Damage Diagnosis of Bending Structures Using Support Vector Machine

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1. Introduction

Structural Health Monitoring

- For maintaining the performance of structures
- For reducing the maintenance cost

For the efficient SHM,

The system of automatic diagnosis

↓

New damage detection method using limited number of sensors

2. Purpose

To establish a new damage detection method

- Using limited number of sensors
- Natural frequency change of a structure
- Support Vector Machine (SVM)

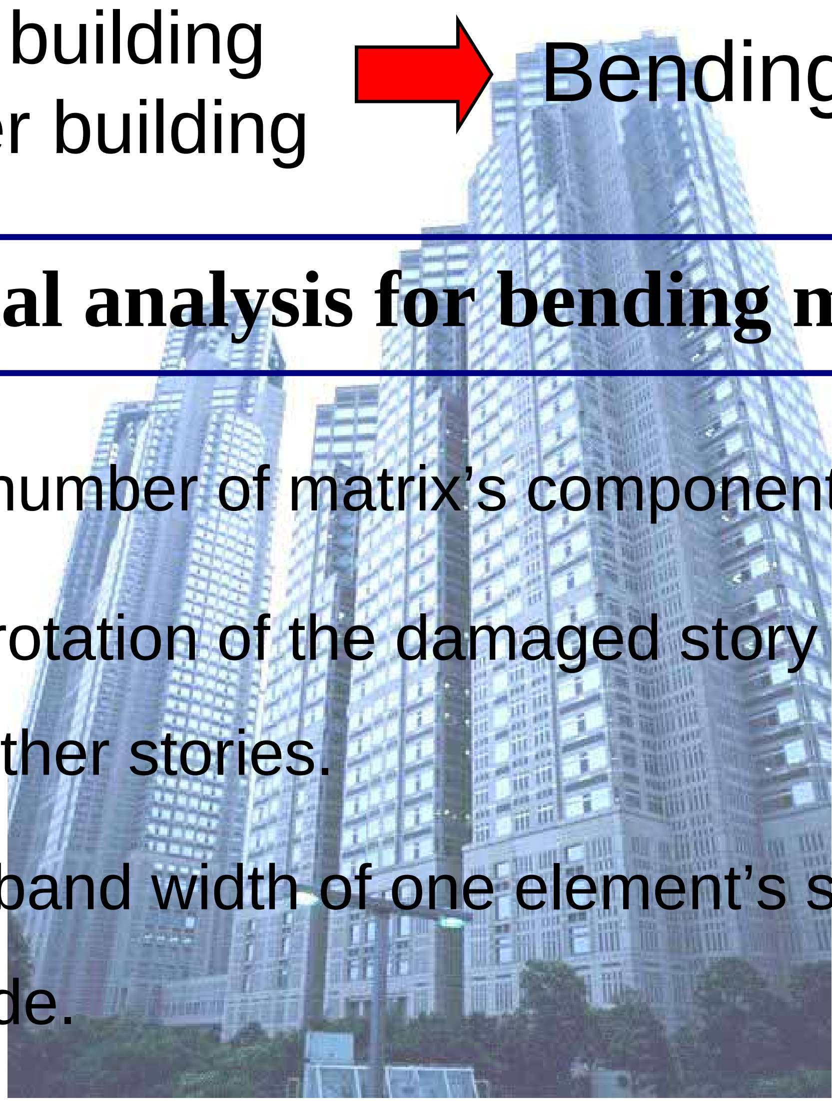
○Shear frame ← only appropriate for low-rise building

⬇

Bending frame ← Tall building

Damage diagnosis of bending structures using SVM

3. Modal Analysis

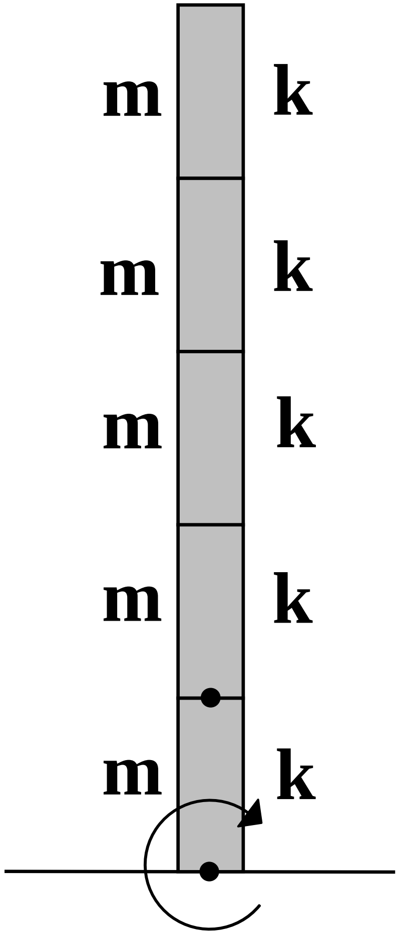


Tall building
Tower building → Bending model

Modal analysis for bending model

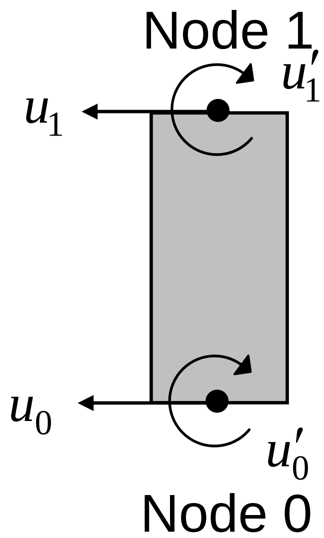
- The number of matrix's components increase
- The rotation of the damaged story influences the other stories.
- The band width of one element's stiffness matrix is wide.

3. Modal Analysis



The matrices of one beam element for FEM

Node displacement vector $\mathbf{u} = \left[u_0 \quad u_0' \quad u_1 \quad u_1' \right]^T$



Mass matrix

$$\mathbf{m} = \frac{\rho A l}{g} \begin{bmatrix} 13/35 & 11l/210 & 9/70 & -13l/420 \\ 11l/210 & l^2/105 & 13l/420 & -l^2/140 \\ 9/70 & 13l/420 & 13/35 & -11l/210 \\ -13l/420 & -l^2/140 & -11l/210 & l^2/105 \end{bmatrix}$$

Stiffness matrix

$$\mathbf{k} = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & -6l \\ 6l & 4l^2 & 6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

Combine matrixes of all elements

3. Modal Analysis

Equation of motion $\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{0}$

By solving the eigenvalue problem,

the modal information can be obtained

Damage **Stiffness reduction**

Natural frequency change

Detect damage using pattern recognition

from patterns of natural frequency changes

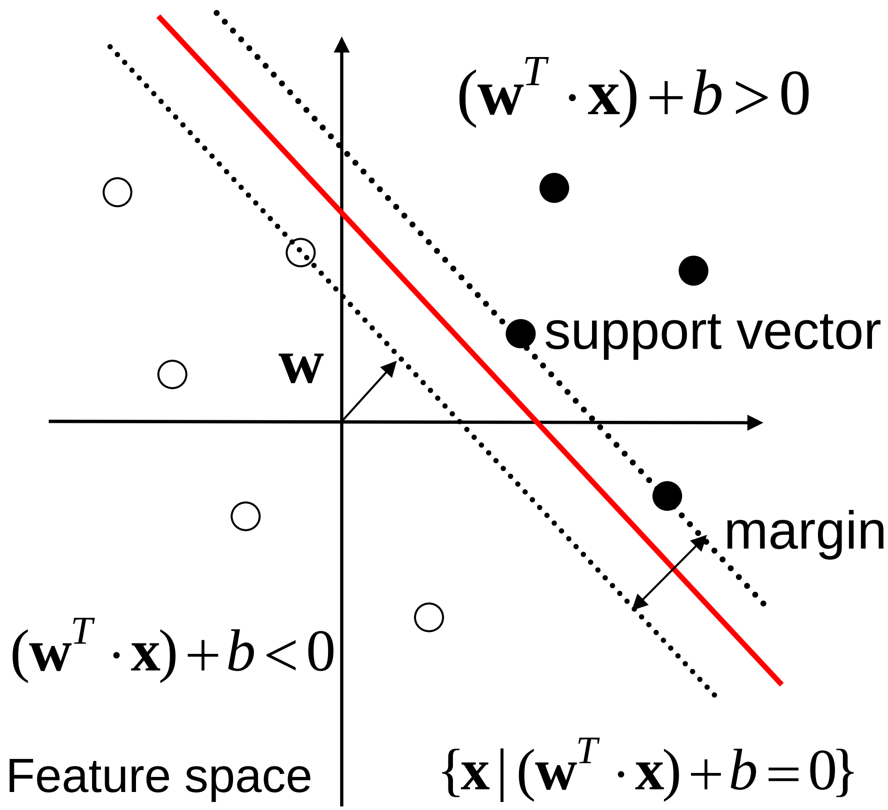
Support Vector Machine

4. Support Vector Machine

The characteristic of SVM･･･**the Maximal Margin Classifier**

A hyperplane of equation $(\mathbf{w} \cdot \mathbf{x}) + b = 0$

Separating hyperplane



Margin

the distance of the discriminate plane and the closest point of the training data

maximize the margin

Optimal hyper plane

5. Proposed Algorithm

Feature vector Rate of natural frequency change

Training data (simulation)

- Undamaged data
- Damaged data ⇐ Stiffness of each story is reduced

Construct 2classes SVM

2classes SVM

- SVM1 : 1st story damaged data and others
- ⋮
- SVMN : Nth story damaged data and others

Input data
(Natural frequency change)

→ SVM1 → 1st story is damaged / 1st story is not damaged
⋮
→ SVMN → Nth story is damaged / Nth story is not damaged

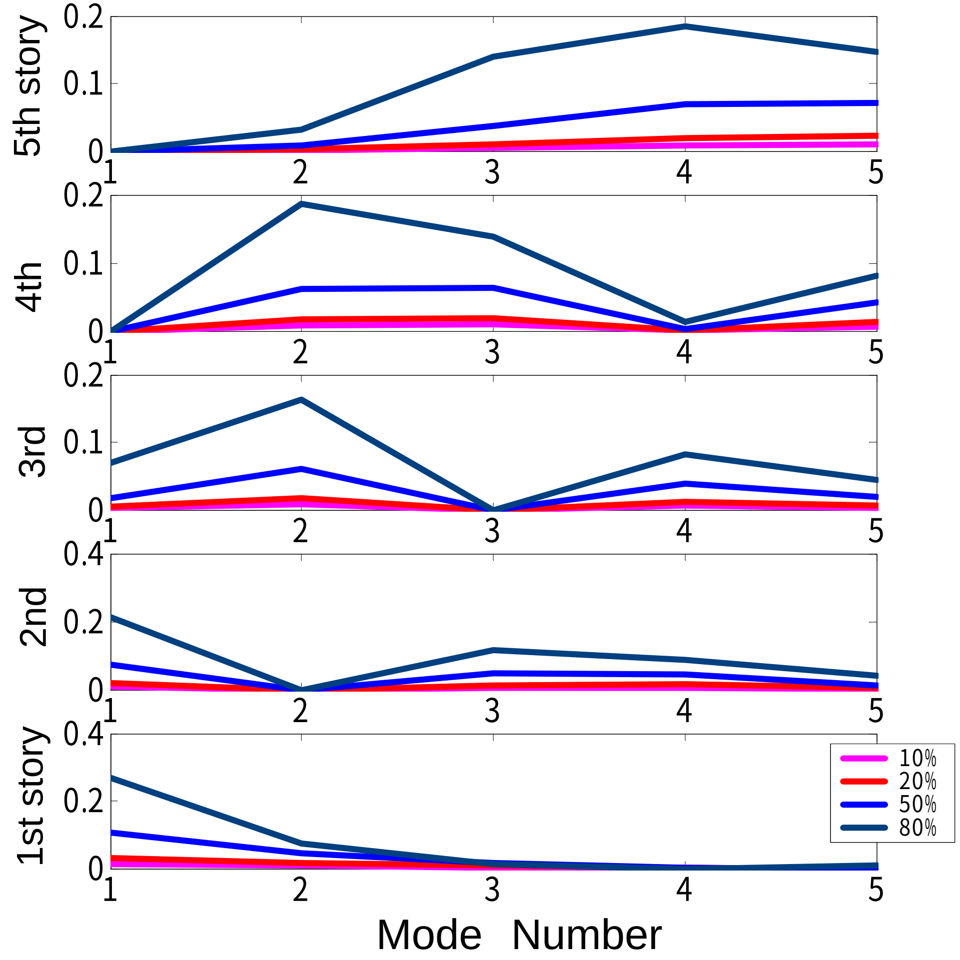
6. Analytical Verification

5-story structure

Feature vector The natural frequency change rate of 1st ~5th mode

The *i*-th story damaged feature vector

$$\boldsymbol{x}_i = \begin{bmatrix} \dfrac{\Delta\omega_{1i}}{\omega_1} & \dfrac{\Delta\omega_{2i}}{\omega_2} & \cdots & \dfrac{\Delta\omega_{5i}}{\omega_5} \end{bmatrix}$$



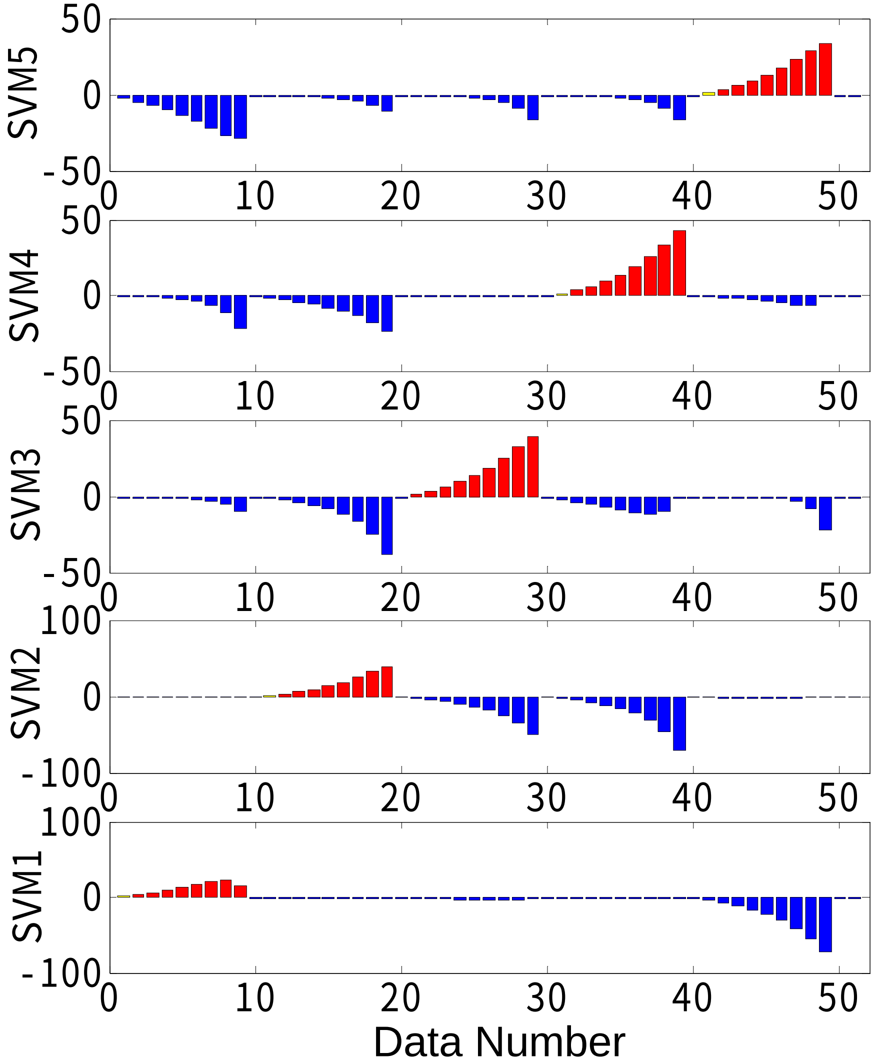
Constructing SVM

Training data :
Stiffness reduction 10%〜90%

Construct SVMs for detecting damage

6. Analytical Verification

Testing SVM using the simulation data



Testing data

No. 1〜9 : 1st story damaged data
No.11〜19 : 2nd story damaged data
No.21〜29 : 3rd story damaged data
No.31〜39 : 4th story damaged data
No.41〜49 : 5th story damaged data
No.10,20,30,40,50 : Undamaged data

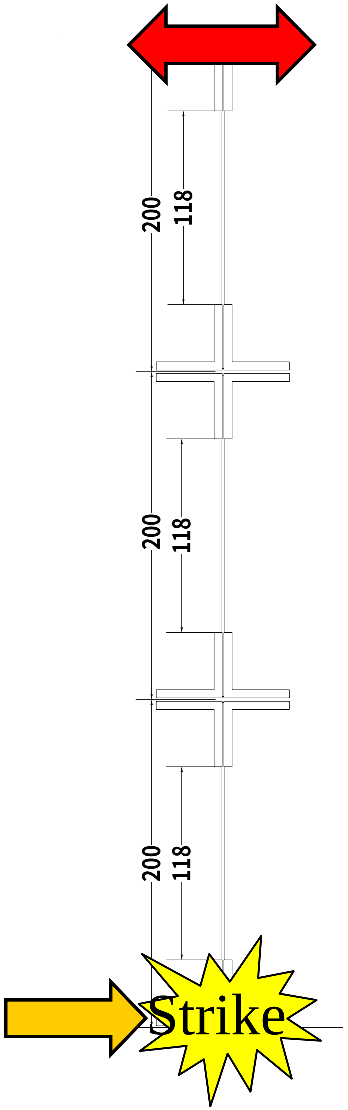
Output of SVM

Output ＞ 0⇒ Damaged
Output ≦ 0⇒ Undamaged

Identify the damaged story by using SVM

7. Experimental Verification

Experimental equipment



3 or 4 story bending model structure

Height of one story : 0.2m

Bronze plate spring

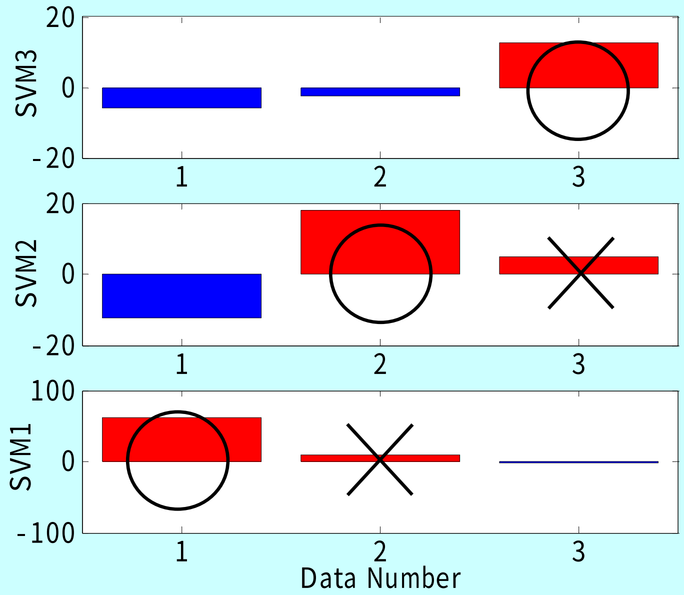
- Undamaged : width 3cm
- Damaged : width 1cm

- Strike the base of equipment
- Measure the acceleration
- Compute natural frequency
- Make the feature vector
- Detect damage by SVM

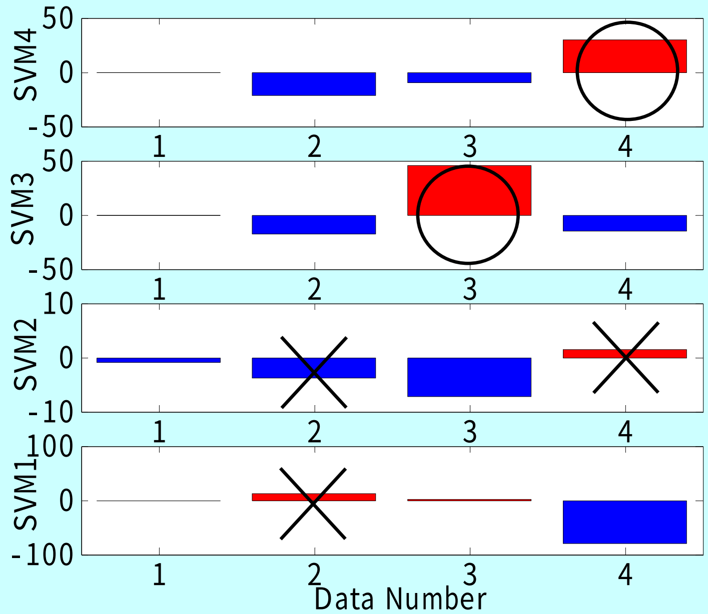
7. Experimental Verification

Verification result

3-story building



4-story building



Damaged story	1st	2nd	3rd	No damage	2nd	3rd	4th

○ ･･･Correct classified data

× ･･･Incorrect discriminated data

➡ Several misclassified data

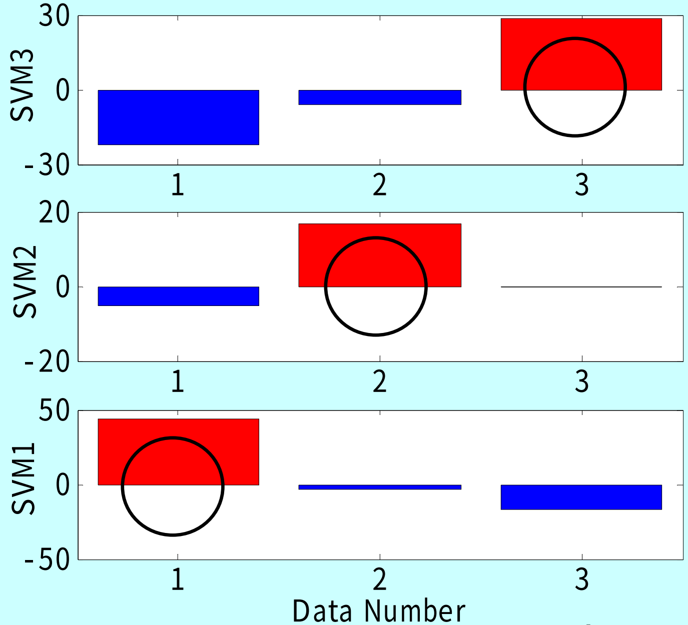
7. Experimental Verification

To improve the classification,

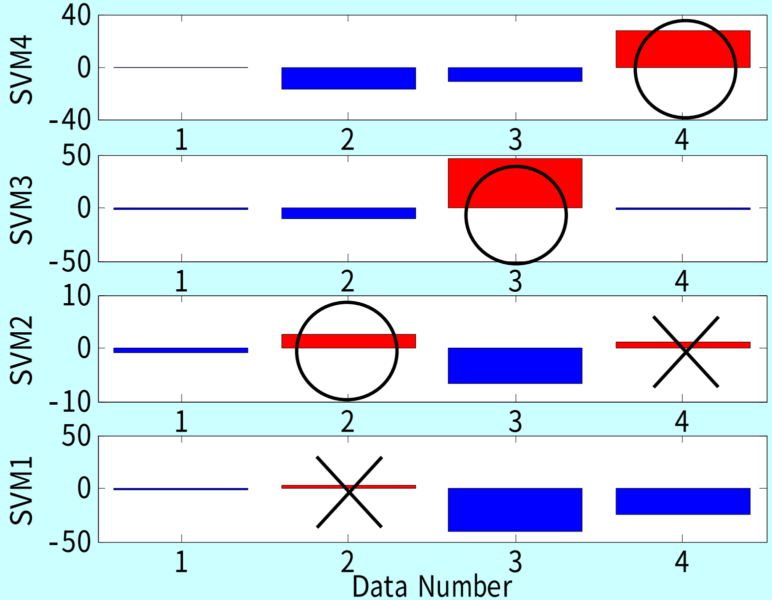
The order of the feature vector was increased

The 1st to 5th modal frequencies were used

3-story building



4-story building



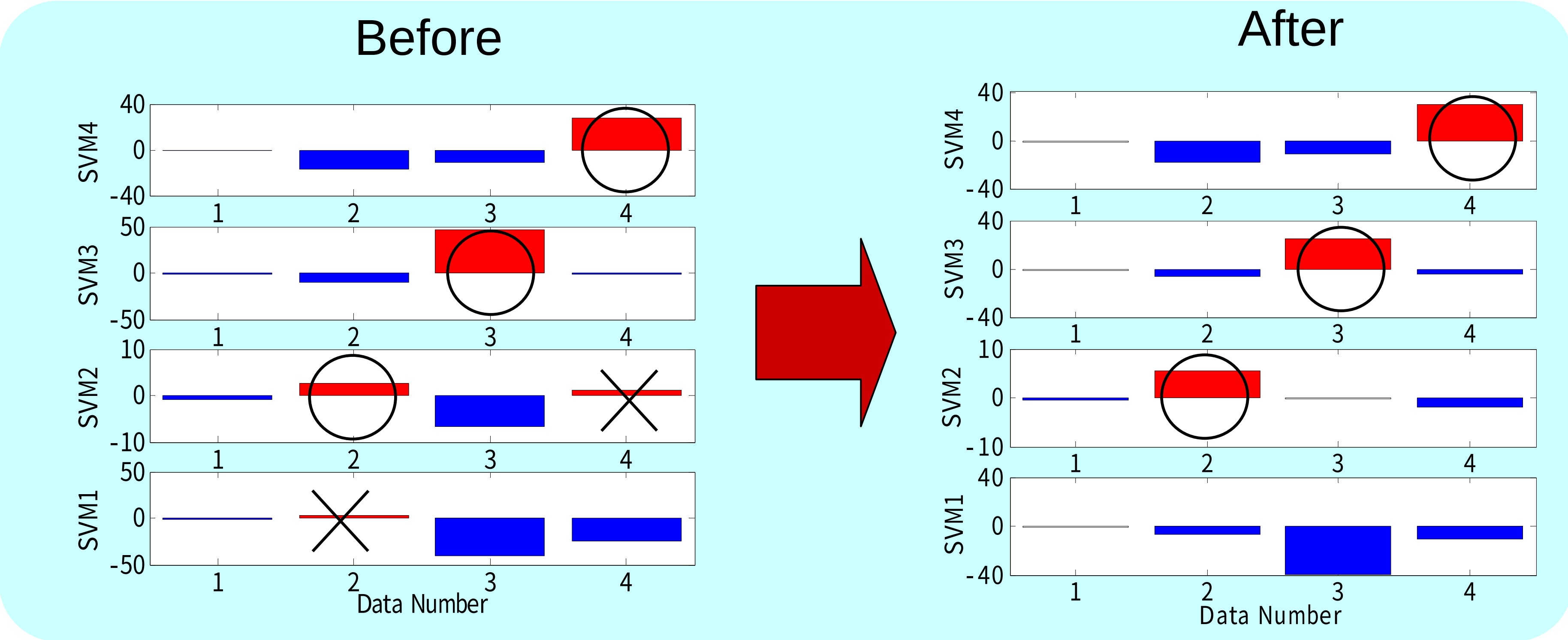
Damaged story 1st 2nd 3rd No damage 2nd 3rd 4th

Misclassified data decreased

7. Experimental Verification

In case of 4-story building,

Updating the simulation model for building SVM



Misclassified data disappeared

8. Conclusion

Simulation of 1-dimensional cantilever model

The damage detection method using SVM is effective for bending structures.

Verification of SVM by experimental data

- The order of feature vector is related to the accuracy of SVM classifier.
- By updating the simulation model, the performance of SVM classifier is improved.

END

Thank you for your kind attention!

0. Outline

4. Support Vector Machine

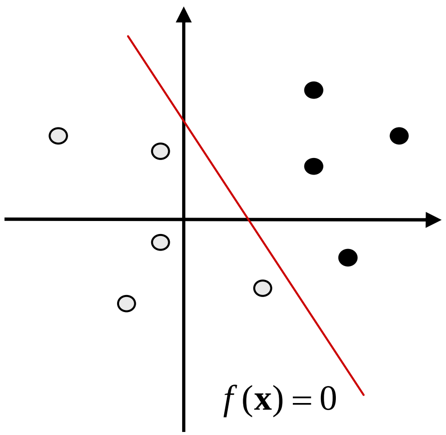
Support Vector Machine (SVM)

- A mechanical learning system
- Introduced by Vapnik and co-workers
- Linear functions in a high dimensional feature space

Linear SVM (LSVM) ･･･The simplest model of SVM

Only for data which are linearly separable

in the original feature space



⇨ Can’t be applicable to many real-world problem

Nonlinear SVM (NSVM)

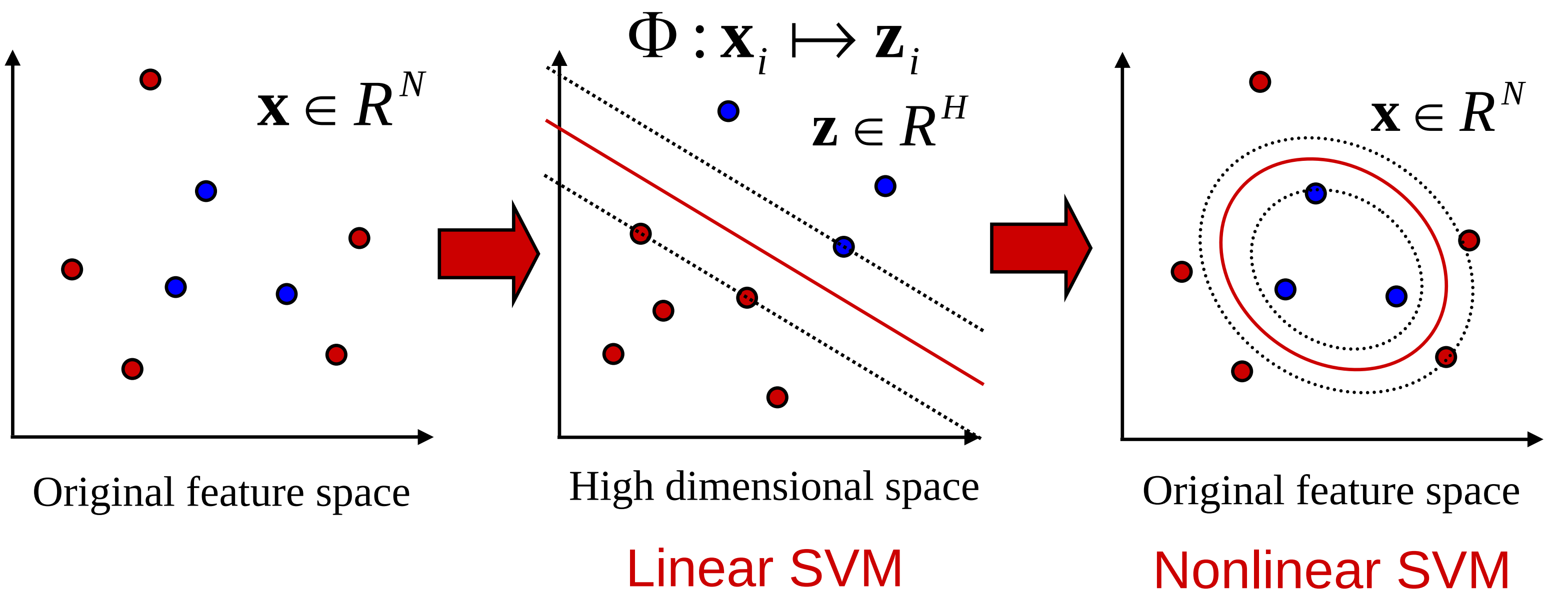
By introducing nonlinear functions called

a Kernel function

4. Support Vector Machine

To allow much more general decision surfaces

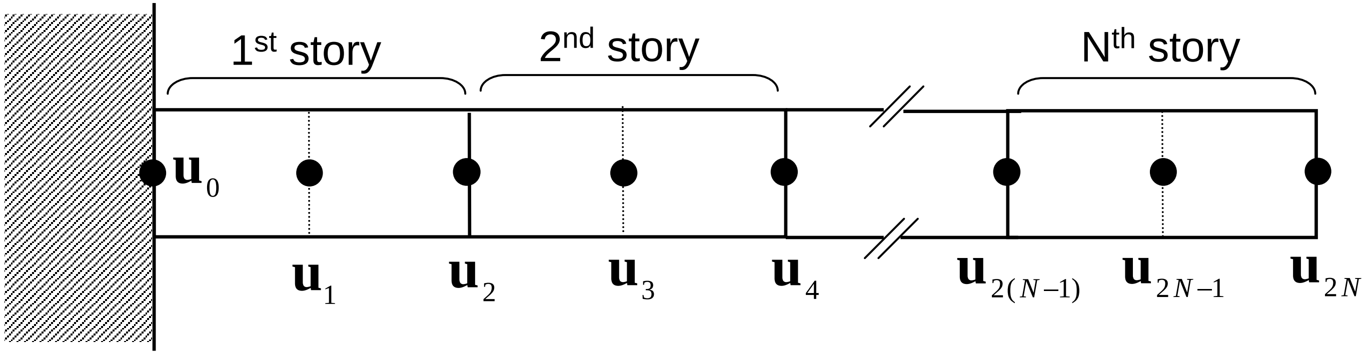
Nonlinear SVM $f(\mathbf{x}) = \mathrm{sgn}(\mathbf{w} \cdot \Phi(\mathbf{x}) + b)$



6. Analytical Verification

Simulation model

1-dimensional cantilever model

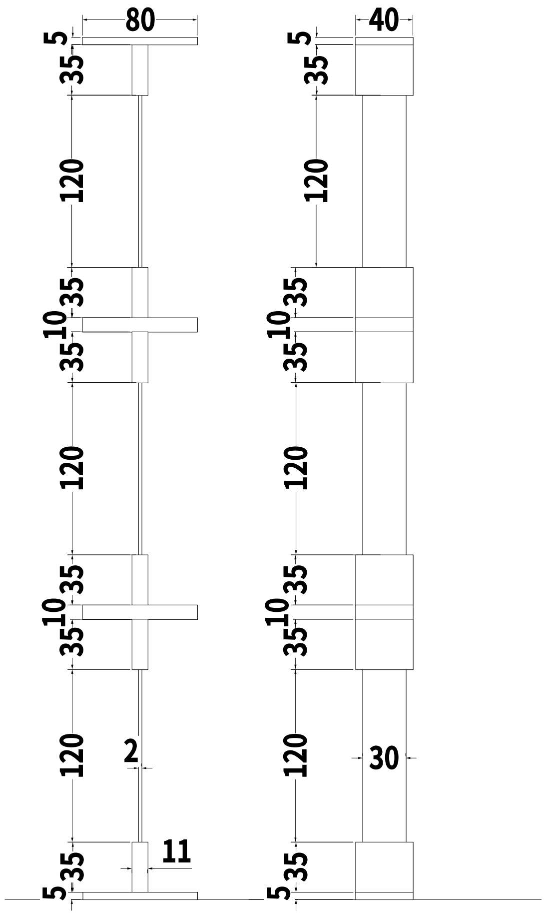


The *i*- th story damaged feature vector

$$\boldsymbol{p}_i = [p_{1i}, p_{2i}, \cdots, p_{Ni}]^T$$

$$= \left[\frac{\Delta\omega_{1i}}{\omega_1}, \frac{\Delta\omega_{2i}}{\omega_2}, \cdots, \frac{\Delta\omega_{Ni}}{\omega_N} \right]^T, \qquad (i = 1, 2, \cdots, N)$$

7. Experimental Verification



実験装置3層

L字金具の接合部分を3つに分ける

実験装置3層

要素数 ⇒ 13

実験装置4層

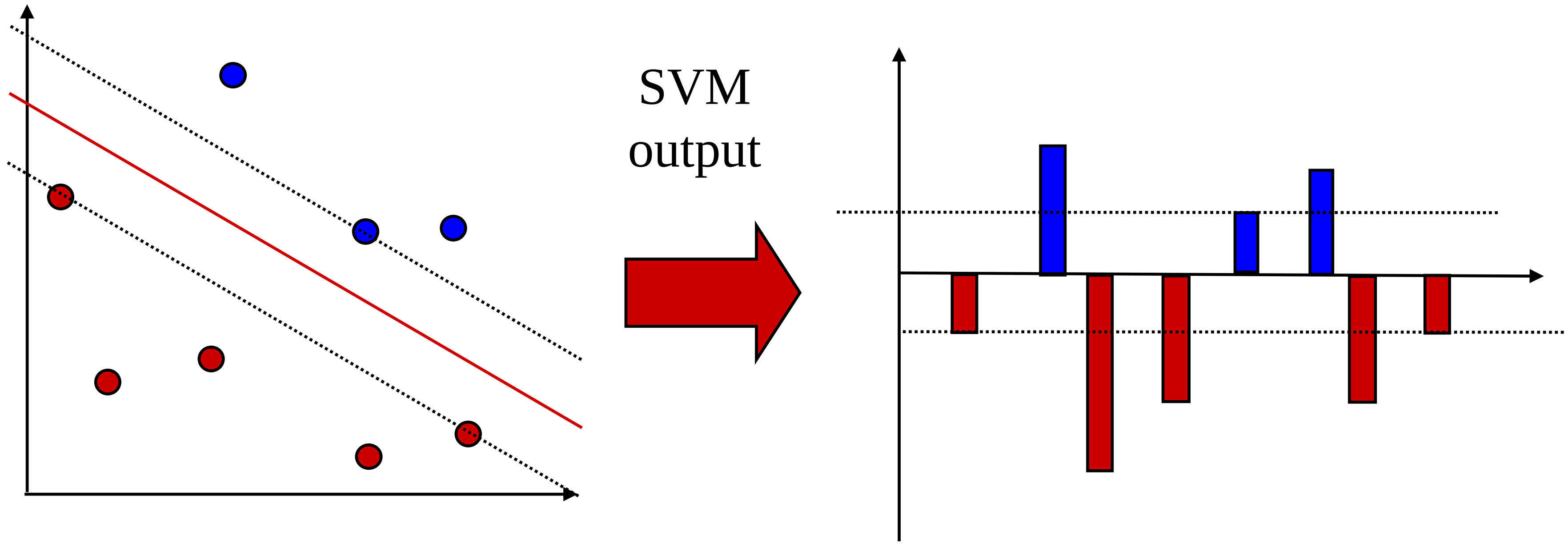
要素数 ⇒ 17

材質

柱 (板ばね) ･･･リン青銅

L字金具 ･･･鋼

4. Pattern Recognition SVM classification

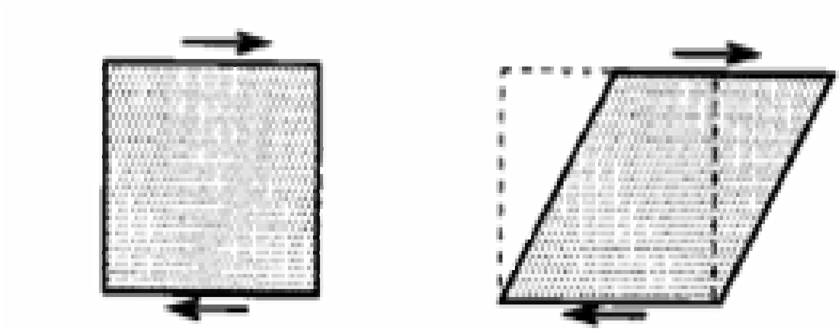


Output of SVM

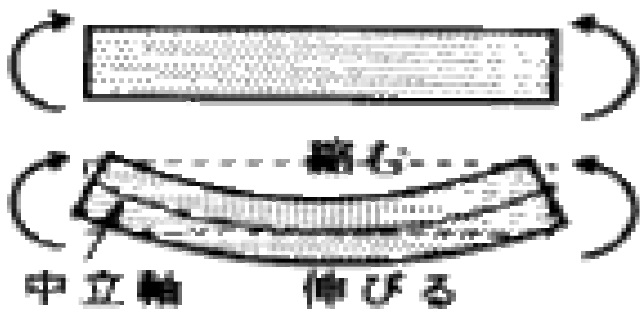
The distance of a data and separating hyperplane

3. Modal Analysis Shear and Bending

shear

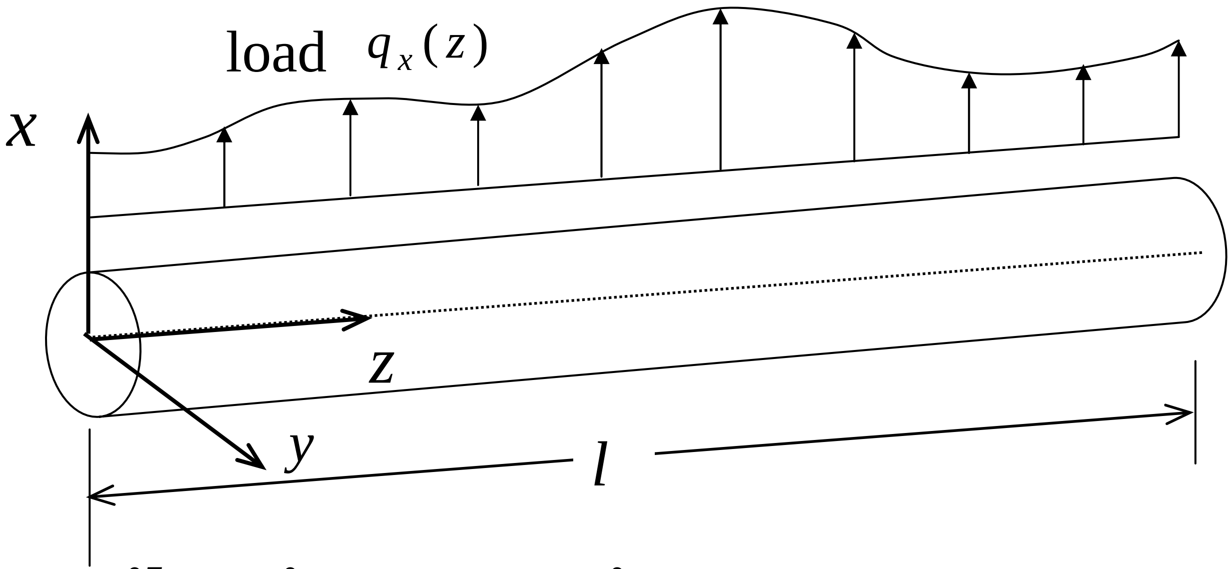


bending



3. Modal Analysis Bending vibration

*Bending vibration analysis for **beam model***



Bending Vibration Equation

$$EI_{xx}\frac{\partial^4 u}{\partial z^4}+\frac{\rho A}{g}\frac{\partial^2 u}{\partial t^2}=q_x(z,t)$$

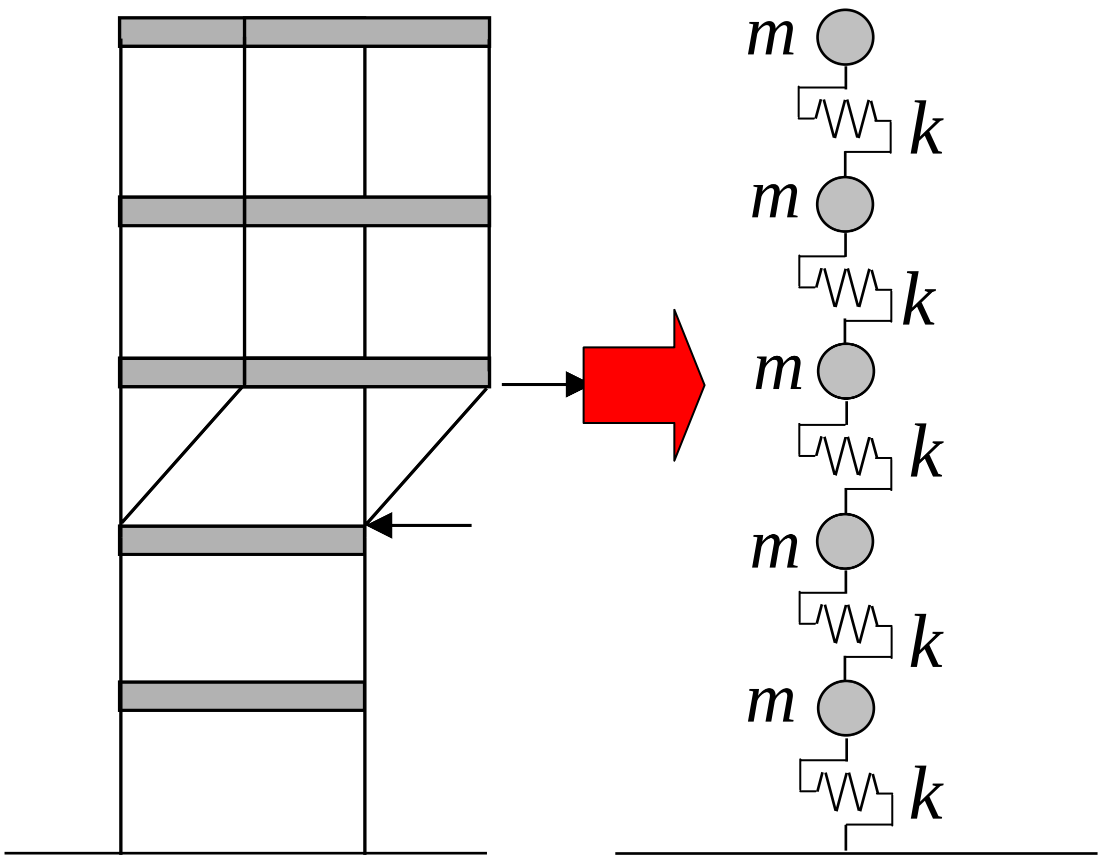
Analysis using Finite element method

3. Modal Analysis   Multi-mass system model

Low building → Multi-mass system model

5-story building

Multiple degrees of system



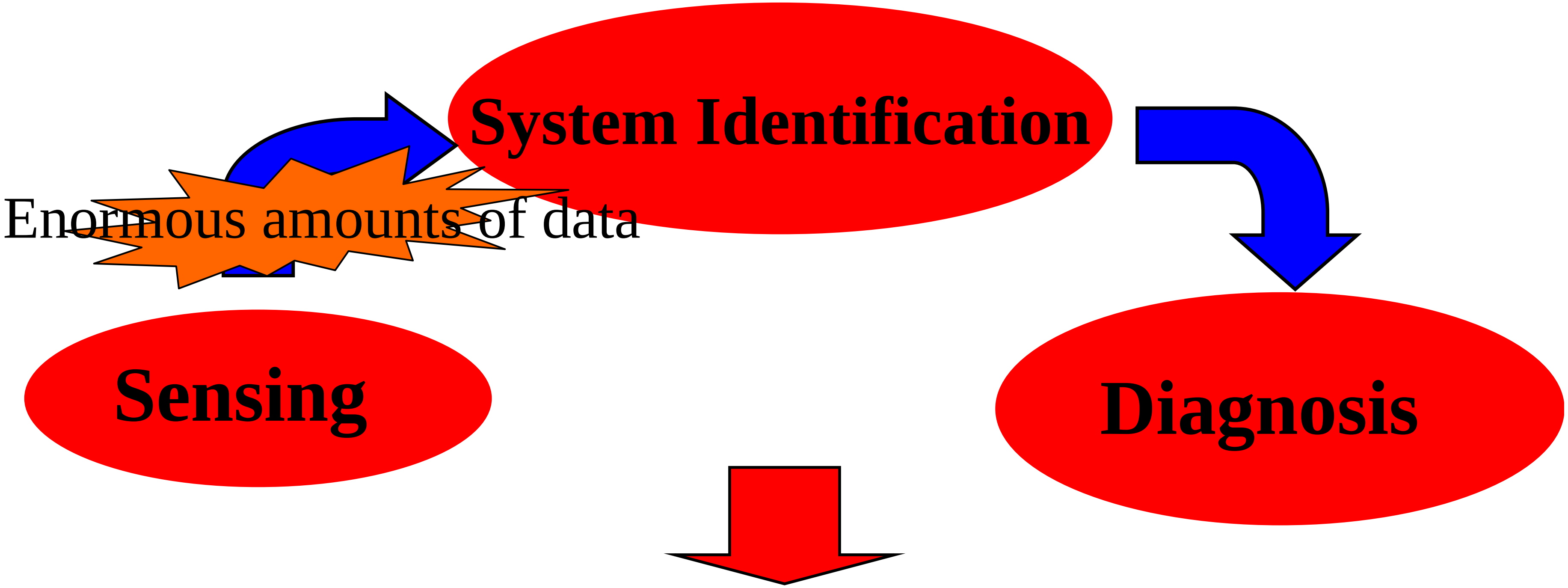
Mass matrix

$$M = \begin{bmatrix} m & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & m \end{bmatrix}$$

Stiffness matrix

$$K = \begin{bmatrix} k & -k & 0 & 0 & 0 \\ -k & k+k & -k & 0 & 0 \\ 0 & -k & k+k & -k & 0 \\ 0 & 0 & -k & k+k & -k \\ 0 & 0 & 0 & -k & k+k \end{bmatrix}$$

1. Introduction Automatic diagnostic system



Extract useful information

and

Diagnosis the damages

Necessity of the automatic diagnosis system

1. Introduction

Necessity of the automatic diagnosis system

⇨ New damage detection method using limited number of sensors

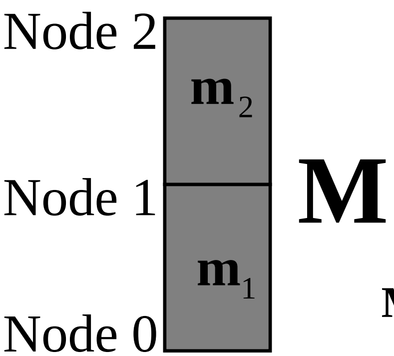
- **Modal frequencies of a structure**
- **Support Vector Machine (SVM)**

Damage Diagnosis for Bending Structures Using Support Vector Machine

3. Modal Analysis Combining matrix

Combining 2 Mass matrixes

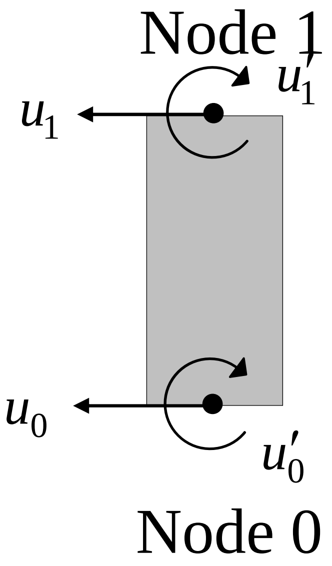
$$\mathbf{m}_1 = \frac{\rho A l}{g}\begin{bmatrix} 13/35 & 11l/210 & 9/70 & -13l/420 \\ 11l/210 & l^2/105 & 13l/420 & -l^2/140 \\ 9/70 & 13l/420 & 13/35 & -11l/210 \\ -13l/420 & -l^2/140 & -11l/210 & l^2/105 \end{bmatrix} \qquad \mathbf{m}_2 = \frac{\rho A l}{g}\begin{bmatrix} 13/35 & 11l/210 & 9/70 & -13l/420 \\ 11l/210 & l^2/105 & 13l/420 & -l^2/140 \\ 9/70 & 13l/420 & 13/35 & -11l/210 \\ -13l/420 & -l^2/140 & -11l/210 & l^2/105 \end{bmatrix}$$



$$\mathbf{M} = \frac{\rho A l}{g}\begin{bmatrix} \frac{13}{35} & \frac{11l}{210} & \frac{9}{70} & -\frac{13l}{420} & 0 & 0 \\ \frac{11l}{210} & \frac{l^2}{105} & \frac{13l}{420} & -\frac{l^2}{140} & 0 & 0 \\ \frac{9}{70} & \frac{13l}{420} & \frac{13}{35}+\frac{13}{35} & -\frac{11l}{210}+\frac{11l}{210} & \frac{9}{70} & -\frac{13l}{420} \\ -\frac{13l}{420} & -\frac{l^2}{140} & -\frac{11l}{210}+\frac{11l}{210} & \frac{l^2}{105}+\frac{l^2}{105} & \frac{13l}{420} & -\frac{l^2}{140} \\ 0 & 0 & \frac{9}{70} & \frac{13l}{420} & \frac{13}{35} & -\frac{11l}{210} \\ 0 & 0 & -\frac{13l}{420} & -\frac{l^2}{140} & -\frac{11l}{210} & \frac{l^2}{105} \end{bmatrix}$$

3. Modal Analysis Finite Element Method (FEM)

1-dimentional finite element bending model



: Deflection

: Angle of deflection

Node displacement vector

$$\mathbf{u} = \begin{bmatrix} u_0 & u_0' & u_1 & u_1' \end{bmatrix}^T$$

l : length of a element

g : acceleration of gravity

ρ : density

E : Young's modulus

A : sectional area

I : moment of inertia of an area

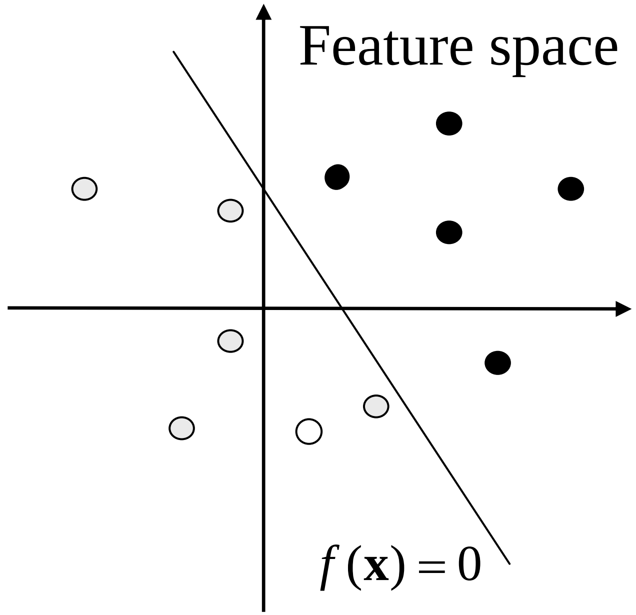
4. Pattern Recognition Basic

Feature extraction

Extract N features from a pattern

N dimensional Feature Vector

$$\mathbf{x} = (x_1, x_2, \cdots, x_N)^T$$



Making a decision rule

Make the decision rule from training samples

Classification

Classify a new pattern to categories according to the decision rule

4. Pattern Recognition Discriminant function

A training sample S

$$S = \left(\left(\mathbf{x}_1, y_1\right), \cdots, \left(\mathbf{x}_N, y_N\right)\right)$$

Feature vector $\mathbf{x}_i \in R^N, i = 1, \cdots, N$ belongs to either of 2 classes

A label y_i ➡ $y_i = \begin{cases} 1 \\ -1 \end{cases}$

Example: 2 categories, A and B

if $y_i = 1$ ➡ $\mathbf{x}_i$ belongs to A

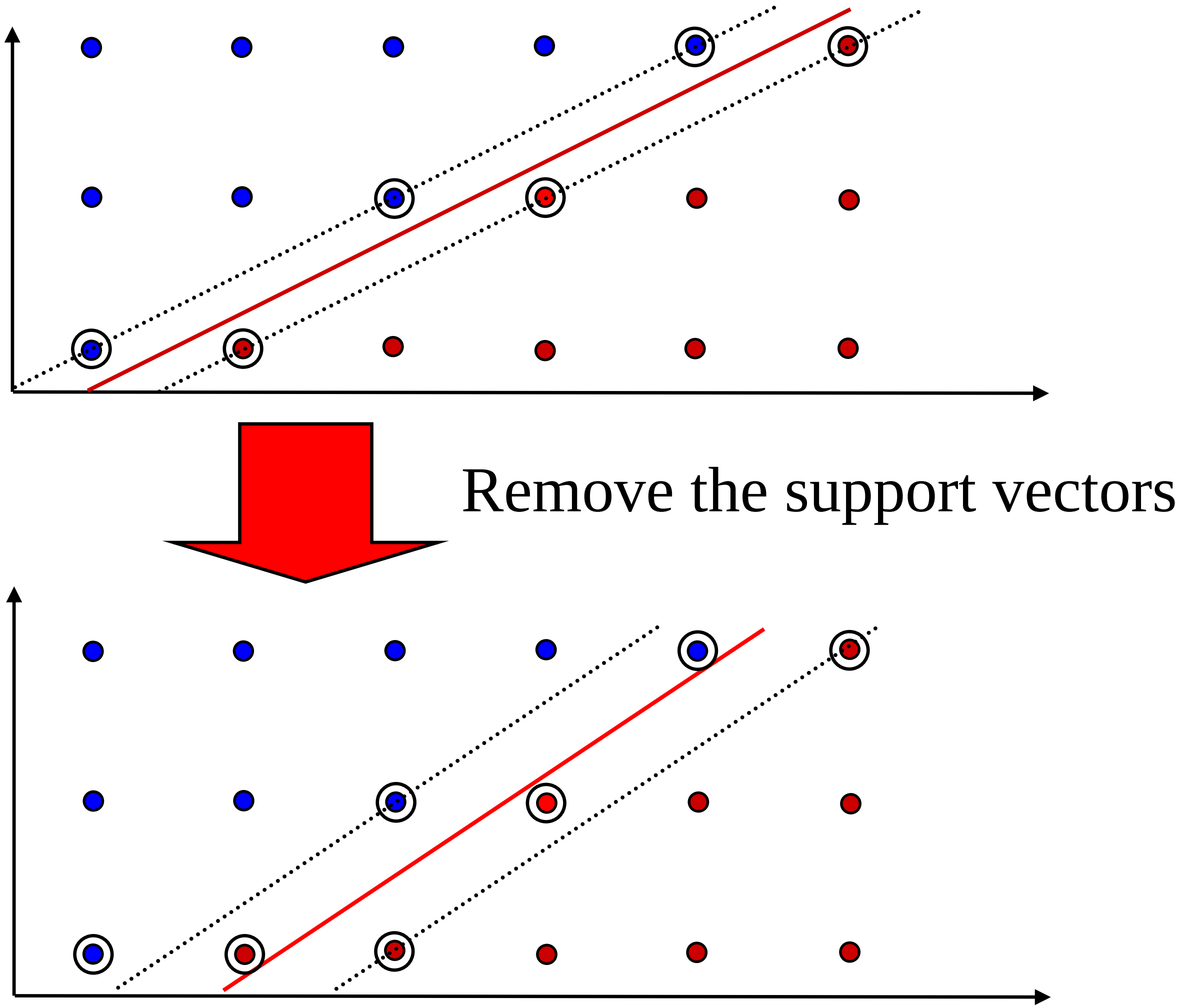
otherwise $(y_i = -1)$ ➡ $\mathbf{x}_i$ belongs to B

Discriminant function of SVM

$$y_i = \mathrm{sgn}\{(\mathbf{w} \cdot \mathbf{x}_i) + b\}$$

$$\mathrm{sgn}[\alpha] = \begin{cases} 1 & \alpha \geq 0 \\ -1 & \alpha < 0 \end{cases}$$

4. Pattern Recognition Support vector



4. Pattern Recognition

Hard Margin SVM

$$y_i = \mathrm{sgn}\{(\mathbf{w}\cdot\mathbf{x}_i) + b\}$$

Maximal Margin (optimization problem)

$$\begin{cases} \text{minimize} \quad f(\mathbf{w}) = \frac{1}{2}\|\mathbf{w}\|^2 \\ \text{subject to} \quad y_i(\mathbf{w}\cdot\mathbf{x}_i + b) \geq 1 \end{cases}$$

In order to relax the situation, allow for a small number of misclassified feature vectors.

Slack variables $\xi_i \geq 0, \quad (i = 1, \cdots, l)$

Soft Margin SVM

$$y_i = \mathrm{sgn}\{(\mathbf{w}\cdot\mathbf{x}_i) + b\}$$

Maximal Margin (optimization problem)

$$\begin{cases} \text{minimize} \quad f(\mathbf{w}, \xi) = \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^{l}\xi_i \\ \text{subject to} \quad y_i(\mathbf{w}\cdot\mathbf{x}_i + b) \geq 1 - \xi_i \end{cases}$$

4. Pattern Recognition Kernel function

Kernel Function

- Radial basis kernel

$$K(\mathbf{x}, \mathbf{x}_i) = \exp\left(-\frac{\|\mathbf{x} - \mathbf{x}_i\|^2}{2\sigma^2}\right)$$

- Polynomial kernel

$$K(\mathbf{x}, \mathbf{x}_i) = (\mathbf{x}, \mathbf{x}_i)^d$$

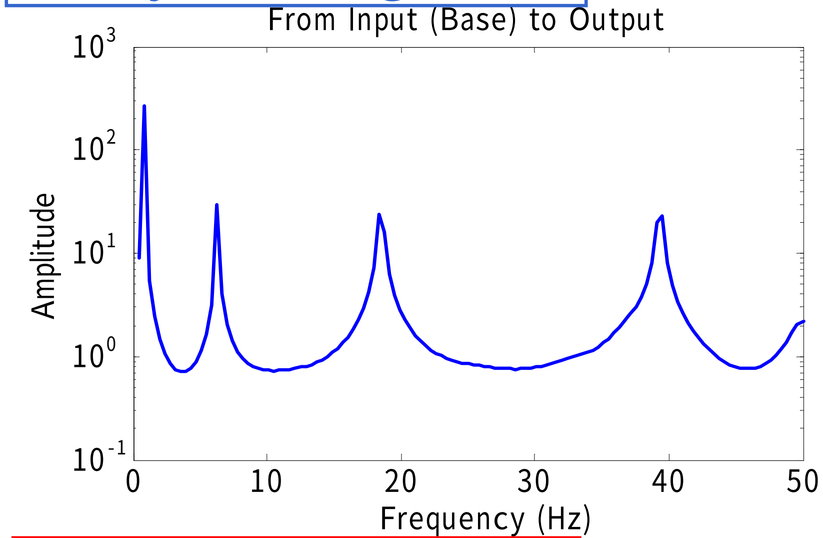
By using Kernel Function,

there is no need
- to compute $\Phi(\mathbf{x})$
- to know the form of $\Phi(\mathbf{x})$

Kernel trick

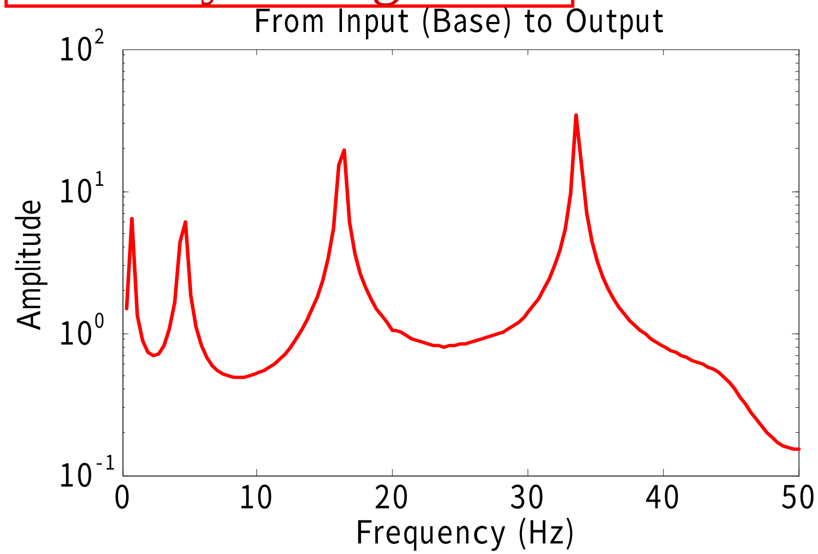
7. *Experimental Verification* Frequency characteristic

4 story undamaged data



Mode	Frequency [Hz]
1st	0.7716
2nd	6.2927
3rd	18.5106
4th	39.2788

3rd story damaged data



Mode	Frequency [Hz]
1st	0.6980
2nd	4.5251
3rd	16.2344
4th	33.6992

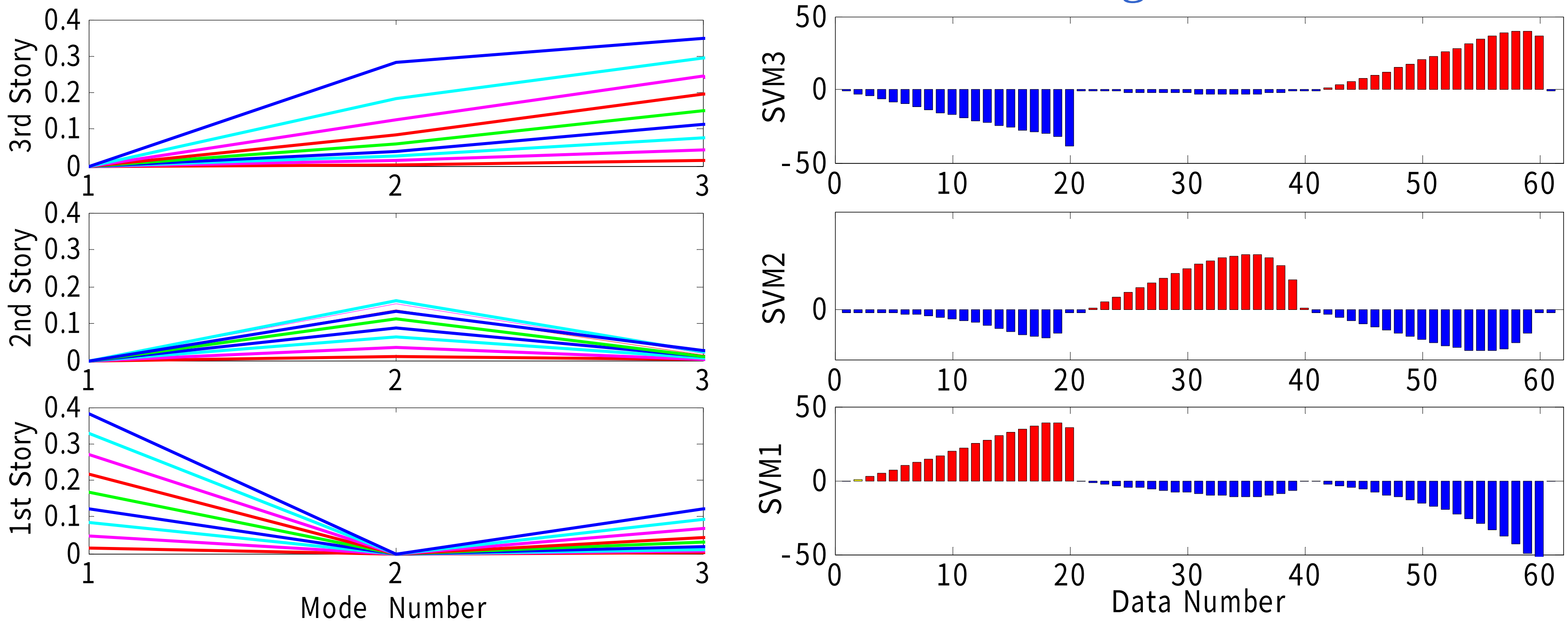
7. Experimental Verification Feature vector ①

① Dimension of feature vector = story number N

Simulation of 3 story model

Damaged data ⇒

Undamaged data ⇒



Feature vector

SVM output

⇒ Verification using the experimental data

7. Experimental Verification Feature vector

Building SVM

2 types of feature vector

① Dimensions of feature vector = story number N

Feature vector ①

***i*th story damaged feature vector of N story model**

$$\mathbf{x}_i = \begin{bmatrix} \frac{\Delta\omega_{1i}}{\omega_1} & \frac{\Delta\omega_{2i}}{\omega_2} & \cdots & \frac{\Delta\omega_{Ni}}{\omega_N} \end{bmatrix}$$

② Dimensions of feature vector = 5

Feature vector ②

***i*th story damaged feature vector of N story model**

$$\mathbf{x}_i = \begin{bmatrix} \frac{\Delta\omega_{1i}}{\omega_1} & \frac{\Delta\omega_{2i}}{\omega_2} & \cdots & \frac{\Delta\omega_{5i}}{\omega_5} \end{bmatrix}$$

8. Conclusion

As a future subject・・・

Applicable to the complicated boundary condition

For example　　A telegraph pole

- Tall (bending) structure
- Various ground
- Overhead wire
- Transformer

→ **Complicated boundary**

→ **Necessity of the simple damage detection method for such problem**