Equivalent Linearization Approach to Probabilistic Response Evaluation for Base Isolated Buildings

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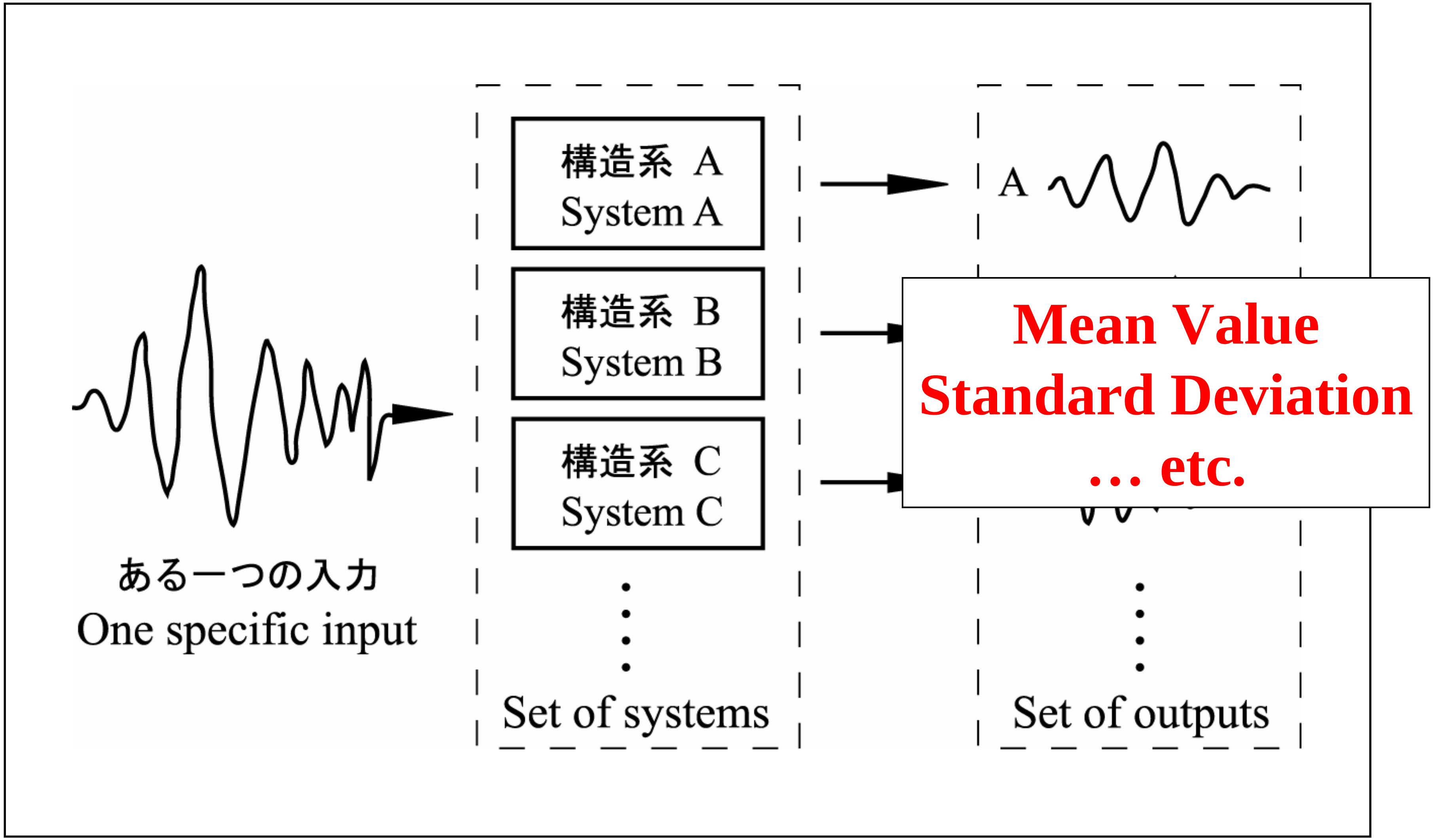
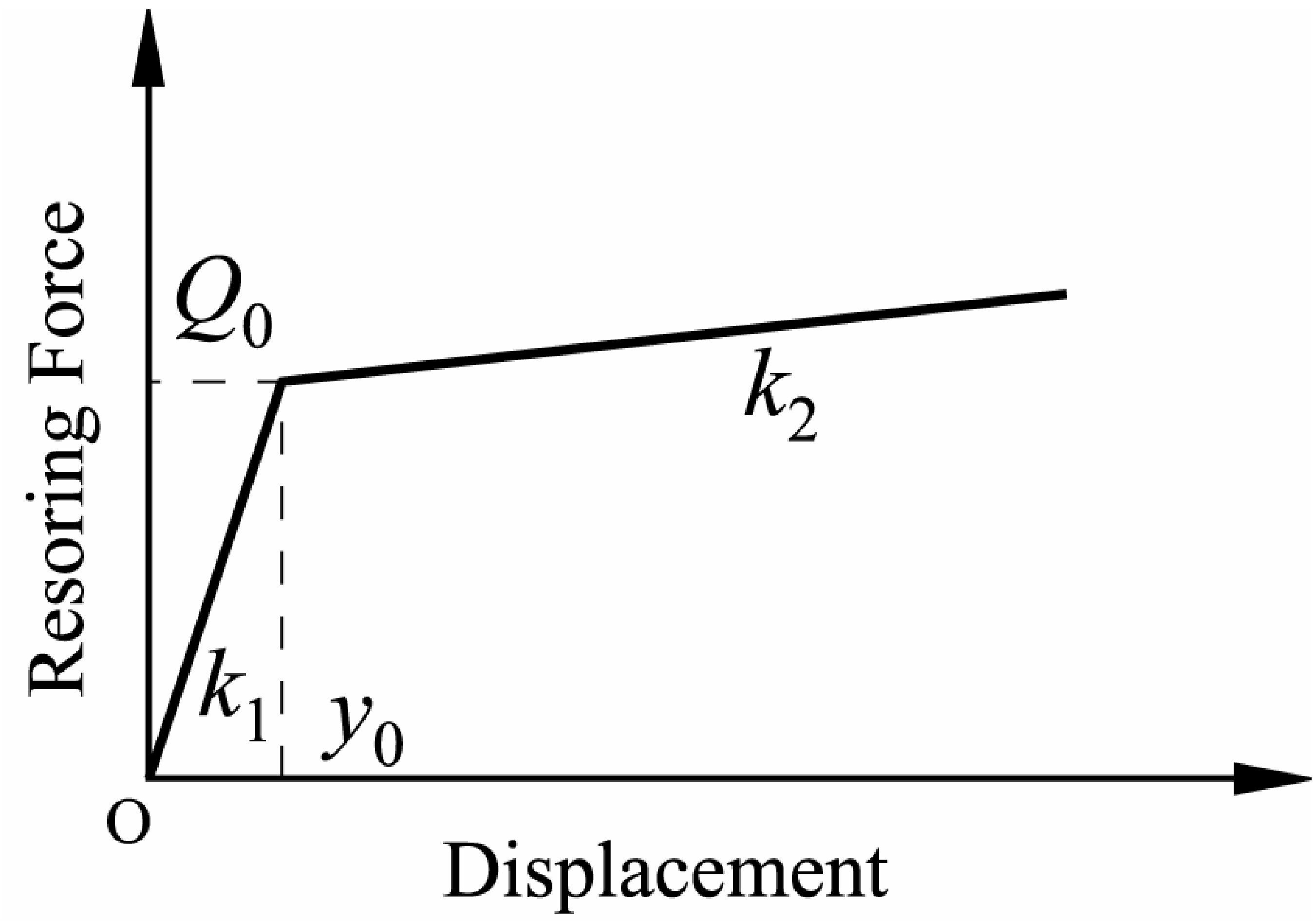
Stochastic Fluctuation involved to Structural System



Base Isolation Scheme with Rubber Bearing and Damper

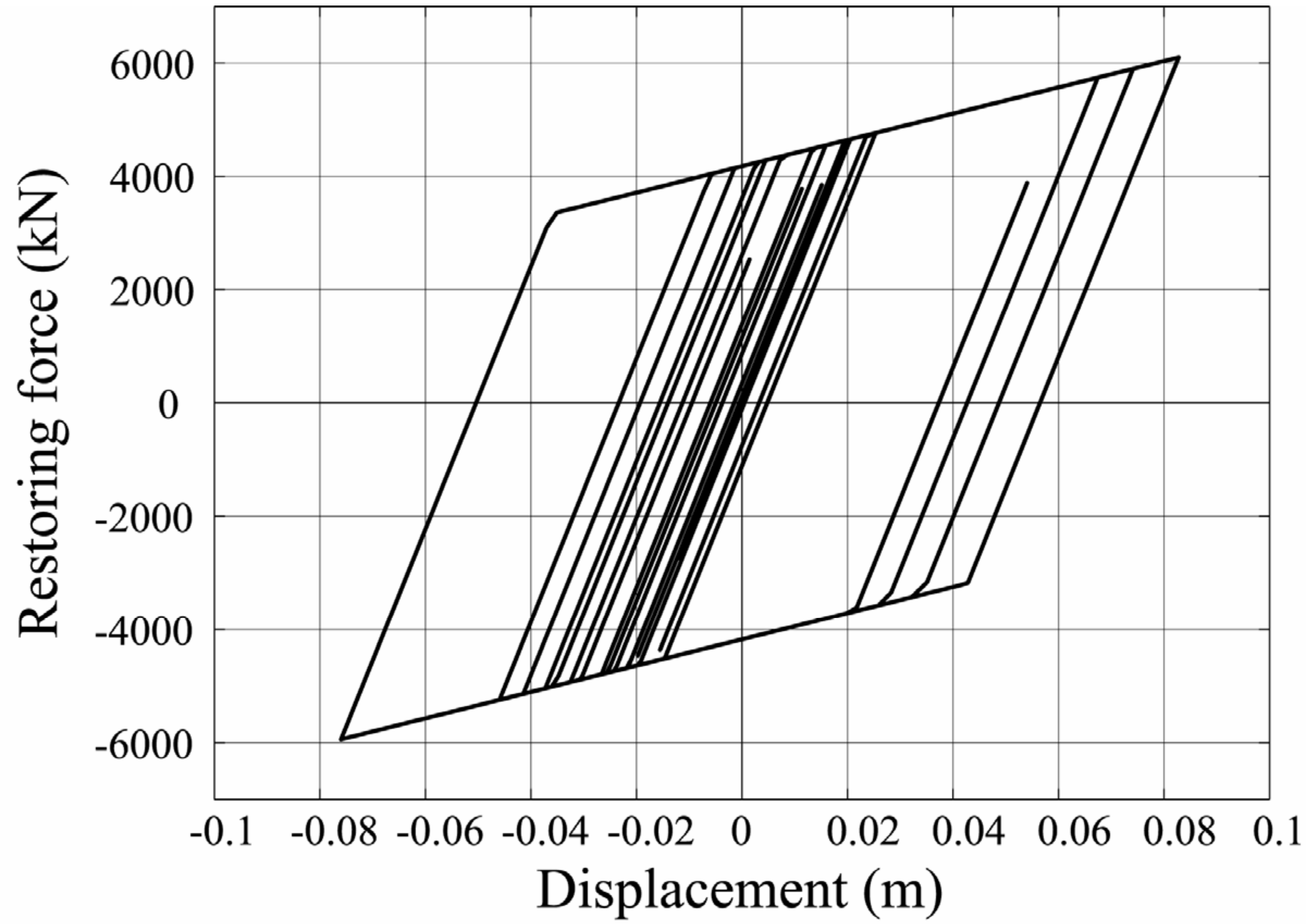
Device of Base-Isolation Scheme (Rubber Bearing)

Relation between Shear of Base-Isolated Layer and Displacement



• Initial Stiffness : k_1 • Second Stiffness : k_2 • Yield Load : Q_0

Bilinear Histeresys for excitation of Random Earthquake



"Expected Structural System"

■ All Parameters are Expected Value itself.

When the System is expressed by …

$$m\ddot{y} + Q\left(y, \dot{y}, k_1, k_2, Q_0\right) = -m\ddot{x}_g$$

"**Expected System**" presents …

$$m\ddot{y} + Q\left(y, \dot{y}, k_1^0, k_2^0, Q_0^0\right) = -m\ddot{x}_g$$

In which …

$$k_1^0 = E[k_1], k_2^0 = E[k_2], Q_0^0 = E[Q_0],$$

(1) Equivalent Linearization Technique

(2) Perturbation Method
(First Order Approximation)

(3) Application Examples

Procedure of First Order Approximation Method

(1) Analysis of "Expected Structural System"
(Equivalent Linearization Technique)

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(2) Evaluation of First Order Variational Ratio

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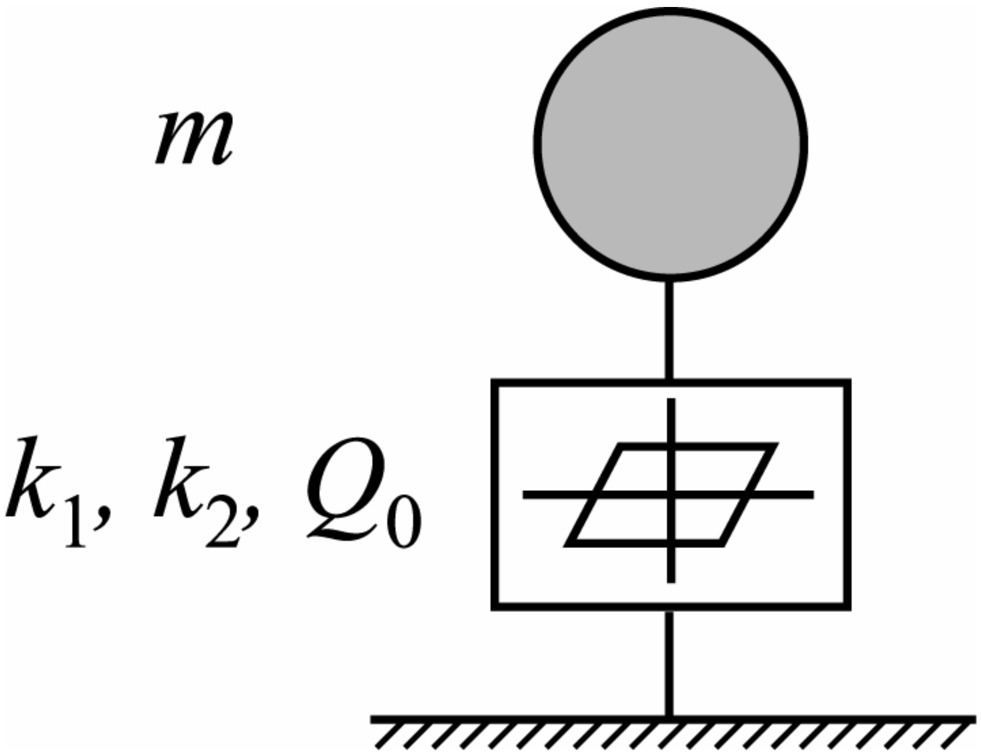
(3) Estimation of Variance (or Standard Deviation)

(1) Equivalent Linearization Technique



Single Degree Of Freedom (SDOF)
Model with Nonlinear Element

Basis of the Equivalent Linearization Technique

$$m\ddot{y} + Q\left(y, \dot{y}\right) = -m\ddot{x}_g$$ ：Nonlinear Govern Equation

$$m\ddot{y} + c_{EQ}\dot{y} + k_{EQ}x + \varepsilon(t) = -m\ddot{x}_g$$

$$\varepsilon(t) = -c_{EQ}\dot{y} - k_{EQ}y + Q(y, \dot{y})$$

c_{EQ} ：Equivalent Damping k_{EQ} ：Equivalent Stiffness

$$m\ddot{y}^* + c_{EQ}\dot{y}^* + k_{EQ}y^* = -m\ddot{x}_g$$

：Equivalent Linearized System

DSM : Dynamic Stiffness Method (by Jennings 1968)

$$\varepsilon(t) = -c_{EQ}\dot{x}^* - k_{EQ}x^* + Q(y^*, \dot{y}^*) = 0$$

Perfect transformation to Linear system

$$\int_{-T/2}^{T/2} \left[-c_{EQ}\dot{x}^* - k_{EQ}x^* + Q(y^*, \dot{y}^*) \right] y^* dt = 0$$

$$\int_{-T/2}^{T/2} \left[-c_{EQ}\dot{x}^* - k_{EQ}x^* + Q(y^*, \dot{y}^*) \right] \dot{y}^* dt = 0$$

Make the “Weighted Residual” to Zero

Equivalent Stiffness and Damping Coefficient by DSM

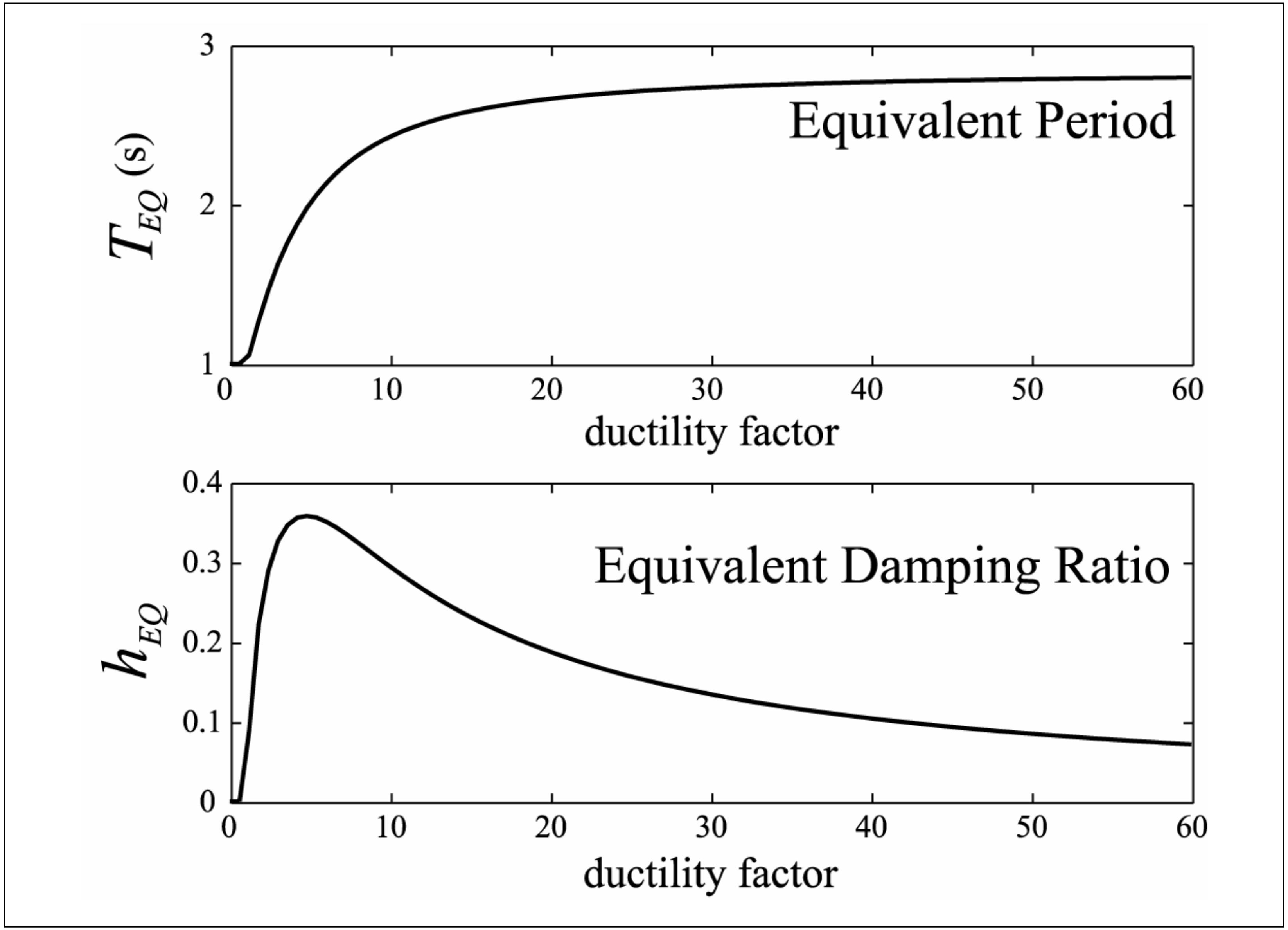
$$k_{EQ} = \frac{1}{\pi}\left[(k_1 - k_2)\theta^* + k_2\pi - \frac{k_1 - k_2}{2}\sin 2\theta^* \right]$$

$$c_{EQ} = \frac{4(k_1 - k_2)}{\pi}\sqrt{\frac{m}{k_{EQ}}}\frac{y_0}{R}\left(1 - \frac{y_0}{R}\right)$$

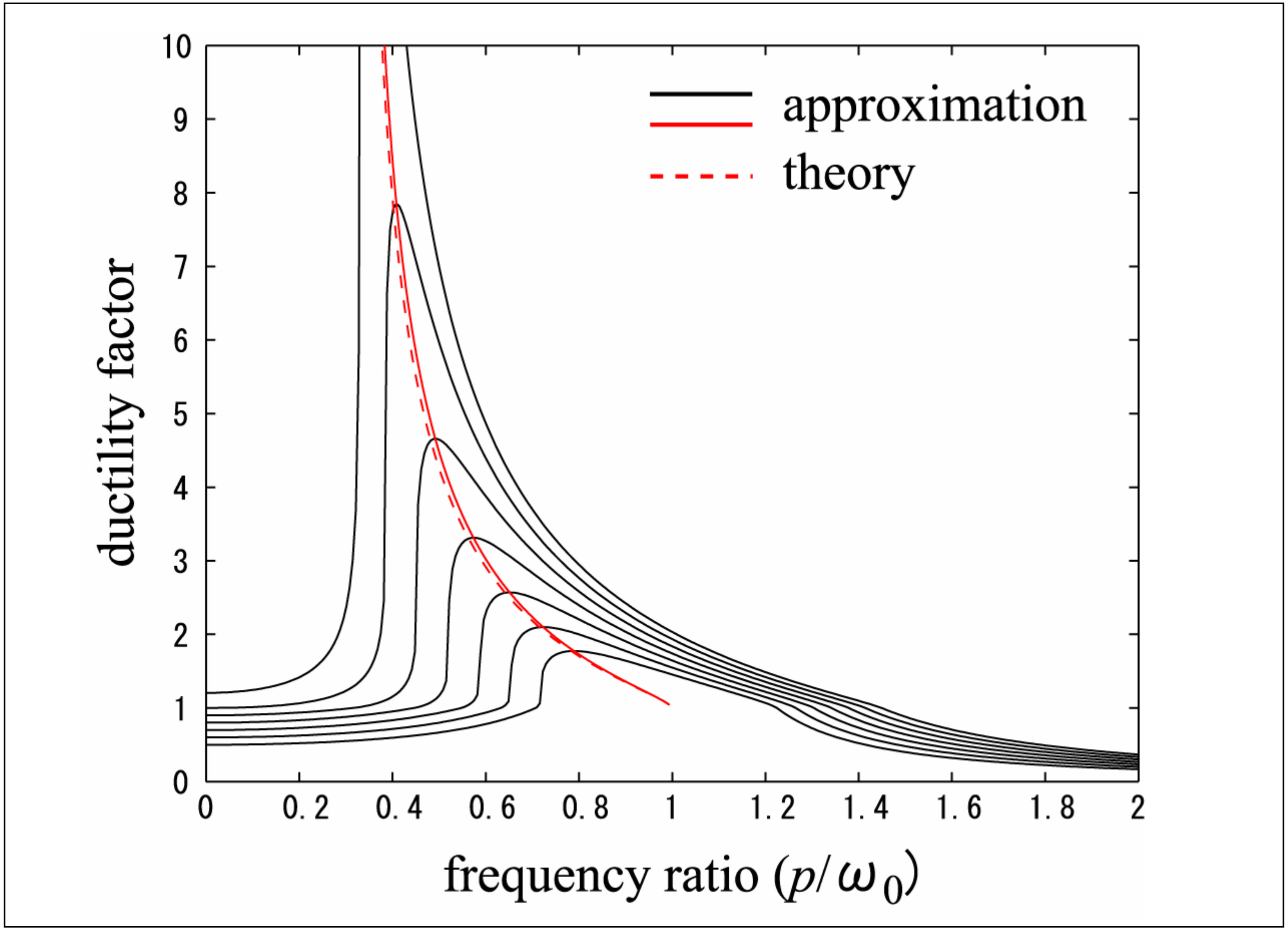
R : Maximu Response $\quad \theta^* = \cos^{-1}\left(1 - \frac{2y_0}{R}\right)$

Note) Those results Consistent with **Caughey**'s method (1960)

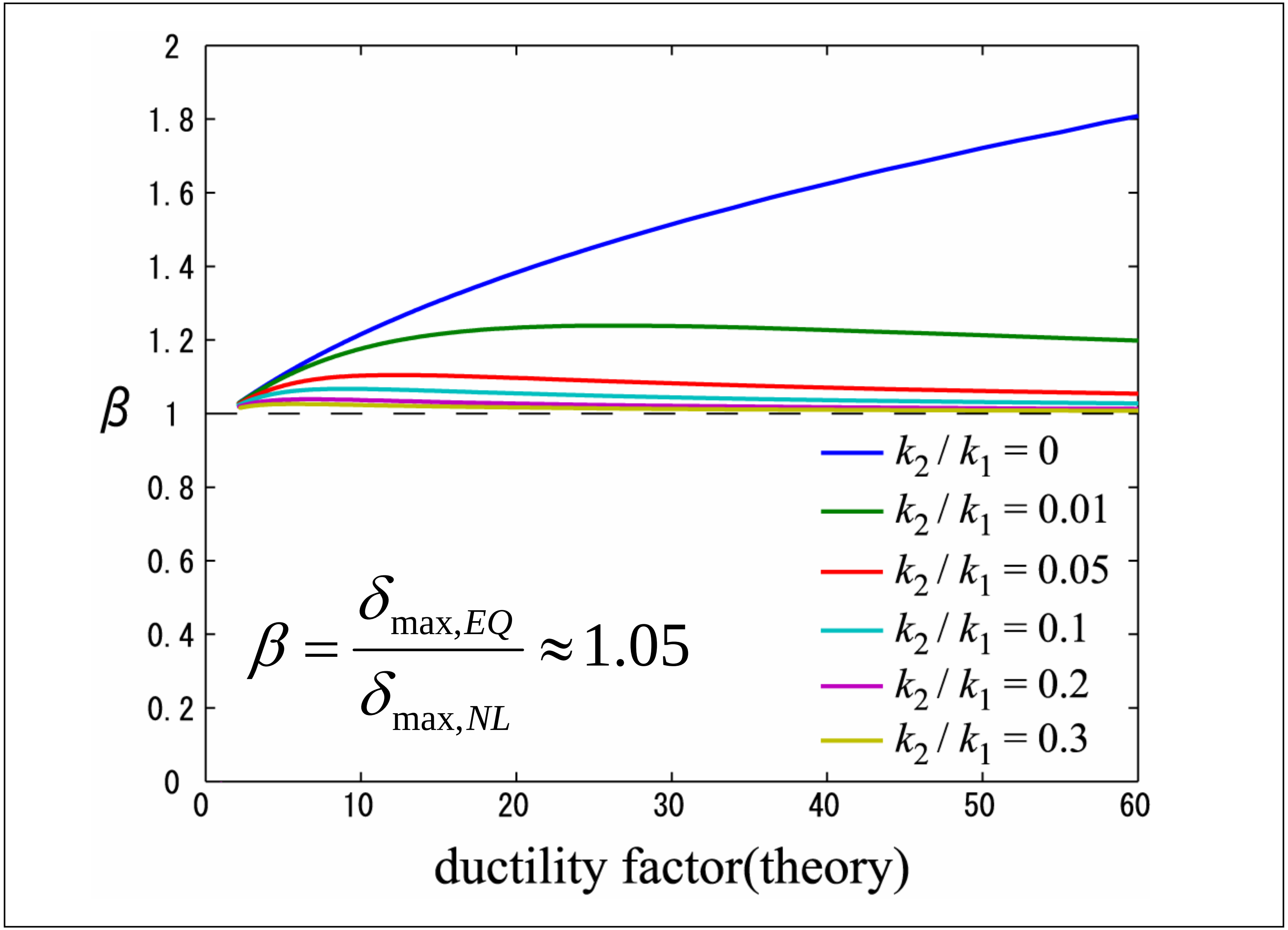
Equivalent Period and Viscous Damping Ratio by DSM



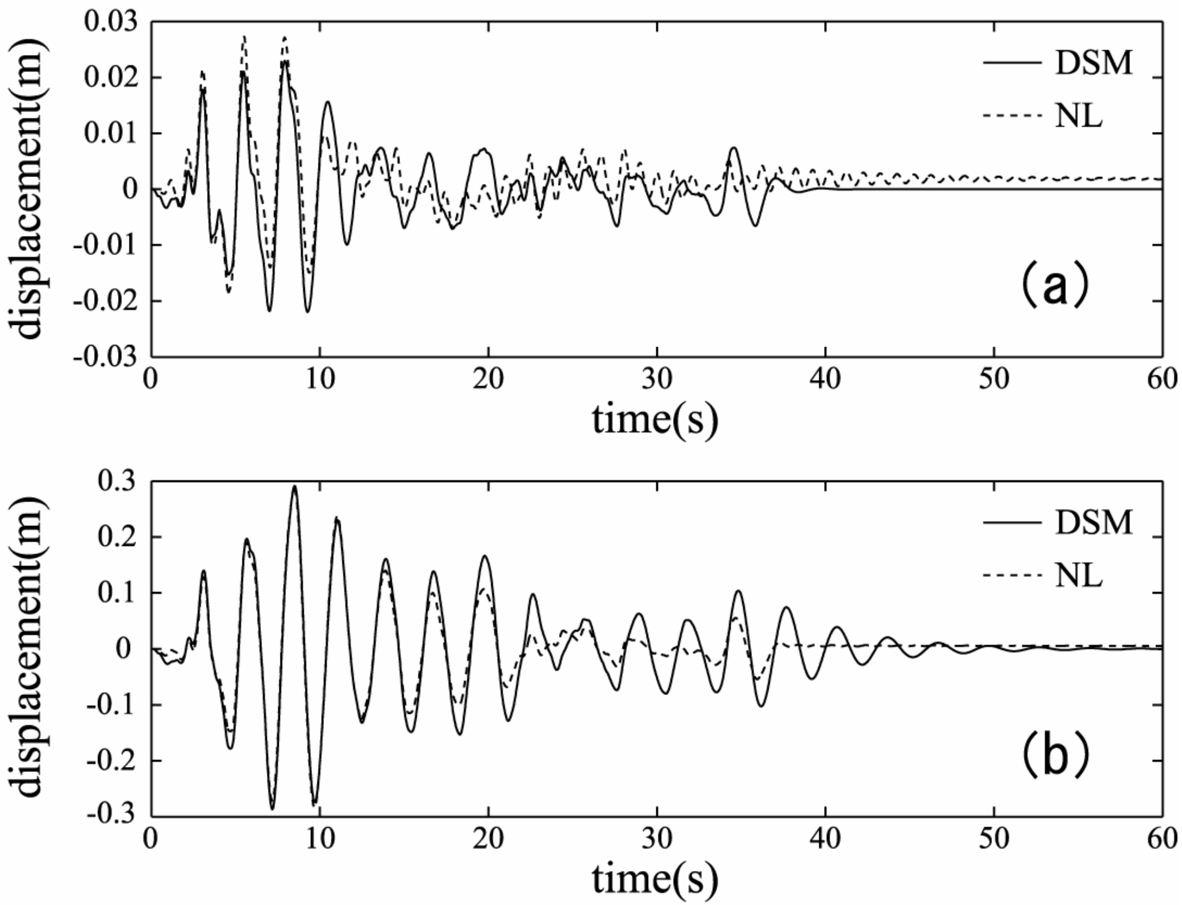
Resonance Curve of Bilinear system (k_2/k_1 = 0.1)



Error of Equivalent Linearization Technique

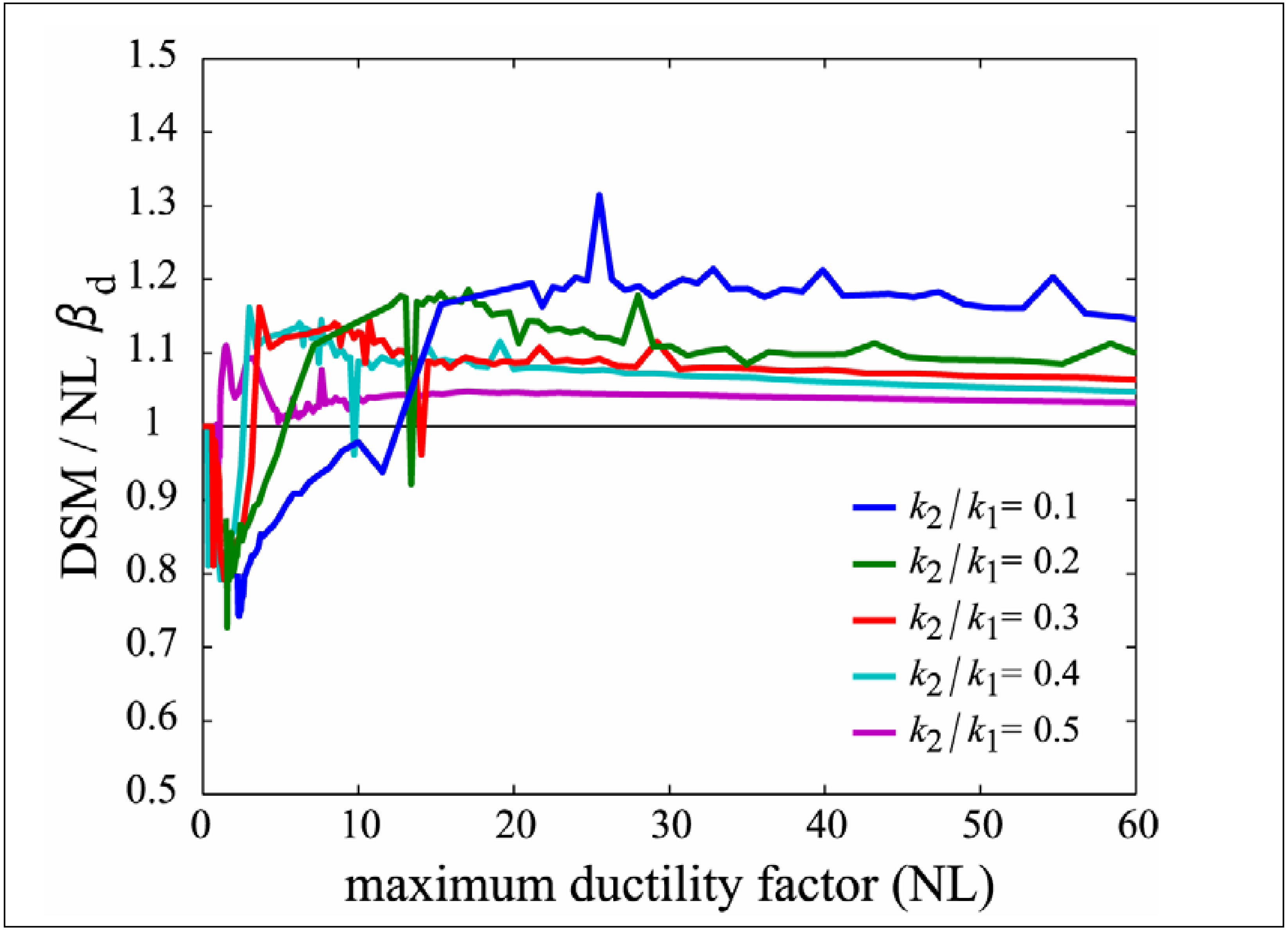


Difference of Accuracy between Small and Large Response



(a) Hachinohe EW, Peak Velocity of **0.1** (m/s)
(b) Hachinohe EW, Peak Velocity of **0.5** (m/s)

In Case of Input of Real Earthquake (Hachinohe EW)



(2) Perturbation Method
(First Approximation Method)

Outline of Perturbation Method

$$D(y,t) = L(y) - f = 0$$ ：Govern Equation
(Equivalent Linearized System)

$$D(y,t) = L^0(y) + R(y) - f = 0$$
：“Expected system” + Stochastic Fluctuation

$$D(y,t) \approx L^0(y) + \sum_{i=1}^{N} \frac{\partial L(y)}{\partial X_i}\left(X_i - X_i^0\right) - f = 0$$

$$y \approx y^0 + \sum_{i=1}^{N} y_i^{\mathrm{I}}\left(X_i - X_i^0\right)$$

Response of “Expected System”

Formulation the Equation determining variational ratio

(i) Equation to determine Zero-order variational ratio ("Expected system")

$$L^0\left(y^0\right) - f = 0$$

(ii) Equation to determine First-order variational ratio

$$L^0\left(y_i^1\right) + \frac{\partial L\left(y^0\right)}{\partial X_i} = 0 \quad i = 1, 2, \cdots, N$$

Calculation of Mean value and Variance

Displacement
Time history

$$y \approx y^0 + \sum_{i=1}^{N} y_i' \left(X_i - X_i^0 \right)$$

Displacement
Mean value

$$E[y] \approx y^0$$

Displacement
Variance

$$Var[x] \approx \sum_{i,j=1}^{N} y_i' y_j' Cov[X_i, X_j]$$

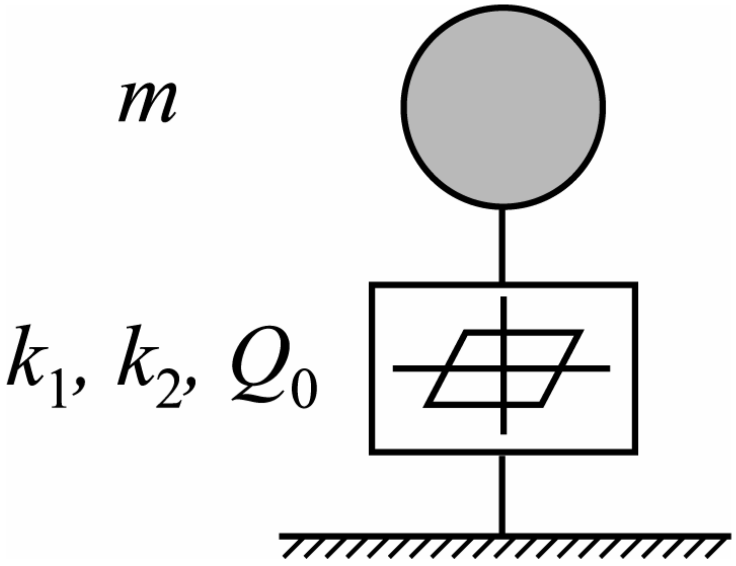
$Cov[\ ]$: Covariance

(3) Numerical Example

Second Stiffness k_2 has Stochastic Fluctuation

with C.O.V 10%

Model Parameters of Base Isolated Buildings



$$m = 1{,}000\,(ton)$$

$$k_1 = 394{,}780\,(kN/m)$$

$$k_2^0 = 0.1 \times k_1$$

$$Q_0 = 0.03 \times mg$$

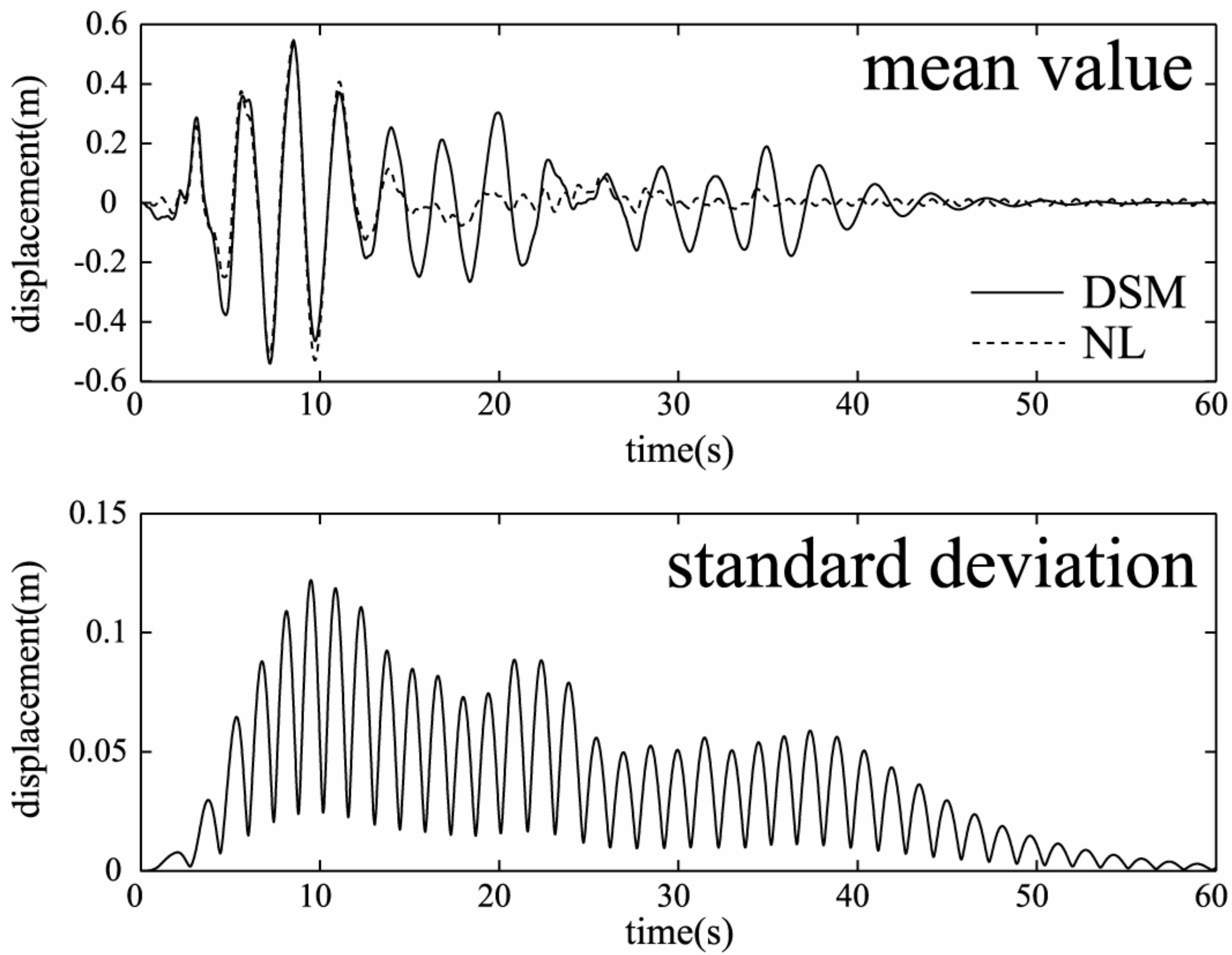
$$= 294(kN)$$

$$k_2 = k_2^0 + \alpha \quad \Rightarrow \quad \mathrm{C.O.V}[k_2] = 10\%$$

Input Load : Hachinohe EW 1968 & JMA Kobe NS 1995

Time Histories of Mean value and Standard deviation

Hachinohe EW 1.0m/s



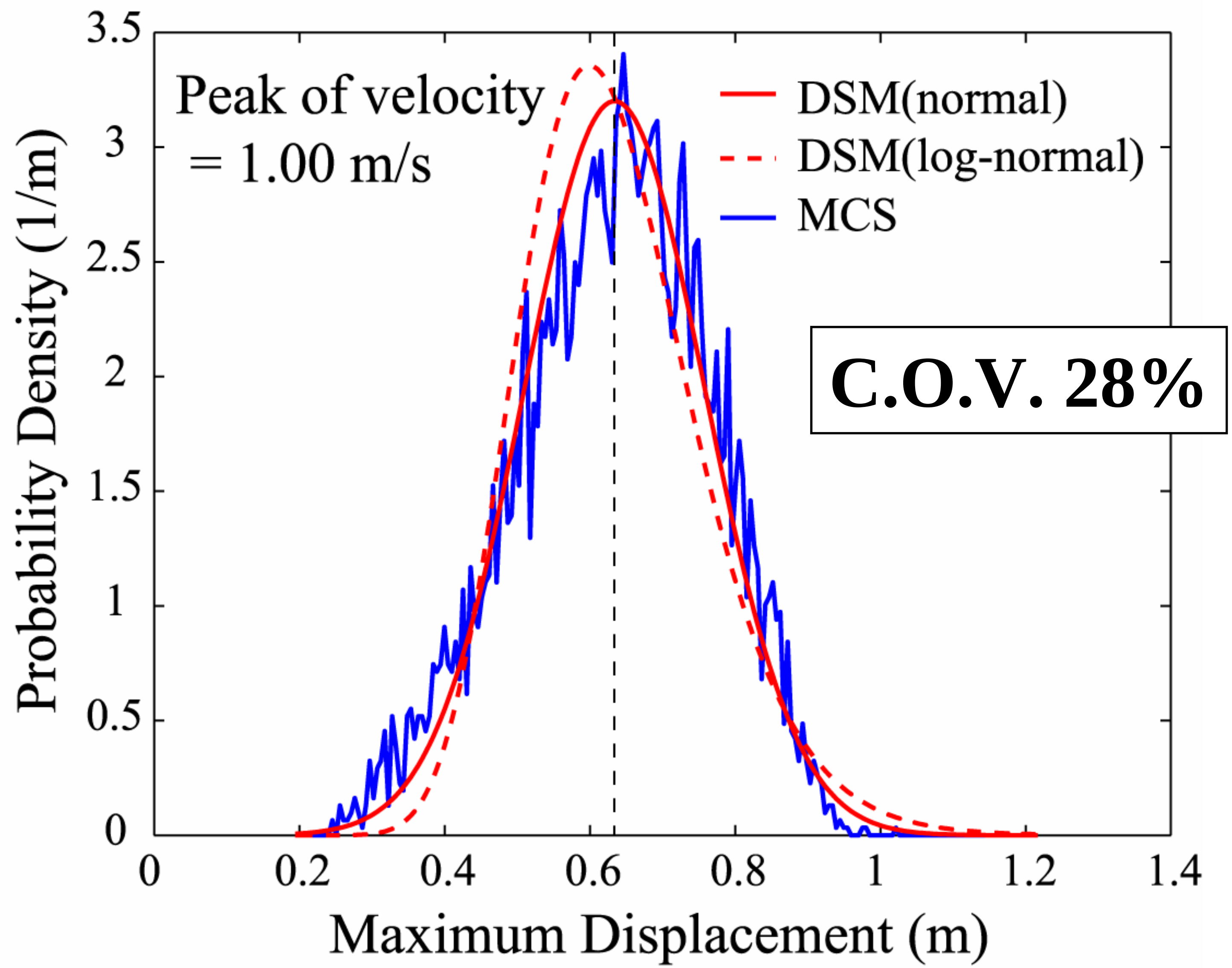
Estimated C.O.V(%) of the Maximum Displacement

Peak velocity (m/s)	Hachinohe (EW)		JMA Kobe (NS)	
	MCS	Perturbation	MCS	Perturbation
0.1	0.26	0.24	0.65	0.07
0.2	2.19	0.44	1.38	0.12
0.3	2.83	1.19	2.43	0.20
0.4	1.17	2.36	3.28	0.31
0.5	4.08	3.99	0.30	0.42
0.6	8.68	5.73	0.41	0.54
0.7	13.47	7.53	0.72	0.67
0.8	14.20	9.25	0.85	0.78
0.9	14.11	11.05	0.77	0.89
1.0	13.55	12.95	0.86	0.99
1.1	13.17	14.74	0.94	1.08
1.2	12.99	16.41	1.05	1.16

(3) Numerical Example

All Parameters have Stochastic Fluctuation.

Hachinohe EW 1.00 m/s (LEVEL 4)



Conclusion

- Perturbation method combined with Equivalent Linearization Technique is applied to base-isolated buildings (SDOF model).

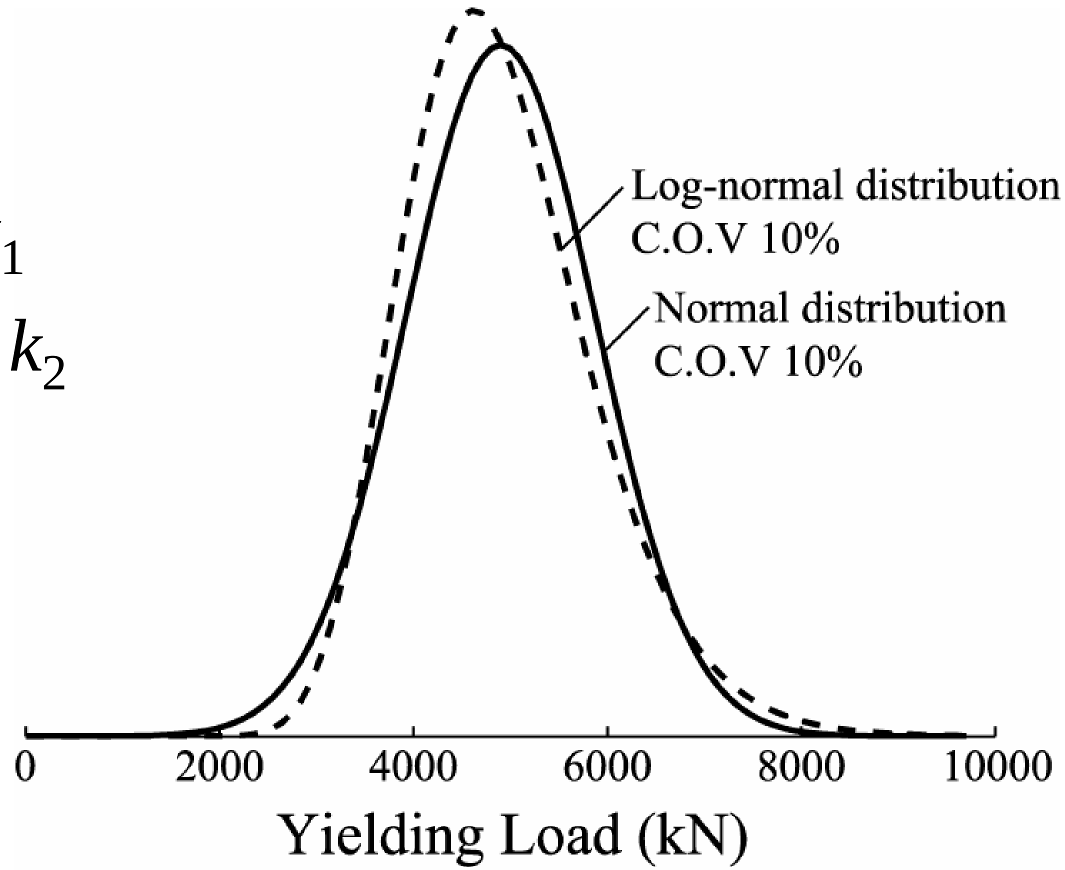
- The expected value and variation of maximum response by proposed method agrees well with result of Monte-Carlo simulation.
  This will be easy and useful method to verify the safety and reliability in the first phase of design.


TOD'S

Stochastic Fluctuation in Material's Property

■ Variable parameters of Base-Isolation system

- Initial Stiffness : k_1
- Second Stiffness : k_2
- Yield Load : Q_0



Summary : Equivalent Linearization Technique (DSM)

(1) Accuracy of Equivalent Linearization Analysis is better in the area of large ductility.

(2) Inversely, Response of linearized system is not stable in small ductility.

$$\frac{\text{Reponse of Analysis for Linearized System}}{\text{Reponse of Analysis for Nonliear System}} = 1.05 \sim 1.1$$