

Equivalent linearization approach to probabilistic response evaluation for base-isolated buildings

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ABSTRACT: The base isolation system for an ordinary base-isolated building consists of rubber bearings and dampers. The system exhibits a nonlinear vibration hysteresis responding to seismic excitation but it is likely to involve fluctuations in its mechanical characteristics. If probabilistically considering such fluctuations in estimating the response of a base-isolated building, nonlinear vibration with probabilistic fluctuations involved has to be dealt with. It would not be an easy task to properly evaluate the response distribution of nonlinear vibrations. In this paper, accounting for the fluctuations involved in the base isolation systems, the statistic response of base-isolated buildings is estimated by utilizing the techniques of equivalent linearization combined with the perturbation approach.

1. INTRODUCTION

During the last two decades, the seismic design of building structures has been significantly and dramatically changing its basic concept and fundamental philosophy. A variety of schemes integrating the concept of “control” or more precisely “response control” have been playing very significant role at the stage of practical structural design and they have been practically applied to a large number of actual buildings. Among those response-control strategies, base isolation is definitely one of the most widely accepted and successfully utilized design schemes. In Japan, in fact, there are far more than 1,000 base-isolated buildings completed as of April 2004. Various kinds of buildings, such as hospitals, city halls, police department buildings, fire department buildings, schools, residential buildings, etc., have employed base isolation system in Japan. The number of base-isolated buildings has significantly increased since the 1995 Kobe earthquake in particular. This earthquake has made Japanese people much more concerned about the seismic safety of buildings.

In a seismically-isolated building, the base isolation system comprises two components: elastomeric bearings consisting of a number of layers of thin rubber and steel plates, and dampers. The former element, with its small horizontal stiffness, brings flexibility into the isolated building and the latter

element dissipates vibration energy to prevent the bearings from exhibiting too large deformations. These two components cooperate in drastically reducing the seismic response accelerations of the upper structure above the isolation system. The isolation system exhibits nonlinear vibration response in which the restoring force cannot be uniquely determined only from its distortion. It is well known that these two components forming base isolation system are likely to involve stochastic fluctuations in their mechanical characteristics. If such fluctuations are taken into account in estimating the response of a base-isolated building, some fluctuation or some probabilistic distribution would result in the fluctuation of the isolated building response. However, it would not be an easy task to properly evaluate the response distribution. Accounting for the fluctuations involved in the base isolation systems, in this paper, the statistic response evaluation of base-isolated buildings is conducted by utilizing the techniques of equivalent linearization with the perturbation approach combined for the purpose of estimating the safety and reliability. One of the additional benefits in employing the equivalent linearization approach is that it can be realized what the stiffness and viscous damping, corresponding to an equivalent linear system, would be like. Such information would help us realize how effectively the base isolation scheme works.

2. EQUIVALENT LINEARIZATION TECHNIQUE

2.1 Basic formulation of equivalent linearization

It would be convenient if a nonlinear dynamical system could be treated as a linear system in estimating its dynamical behavior responding to seismic excitation. Linear systems in general are much easier not only of handling but also of understanding of dynamic characteristics than nonlinear systems. For this reason, equivalent linearization techniques have been utilized at a variety of research and practical design stages for structural engineering. Even today they are still useful and beneficial although modern computer technology make it possible to rapidly conduct a huge amount of complex calculations. In this section, such an equivalent linearization technique, which will be effectively utilized in the succeeding section with the perturbation approach combined, is briefly reviewed. The discussion in this section is mostly based on Caughey's contribution to the linearization treatment (1960).

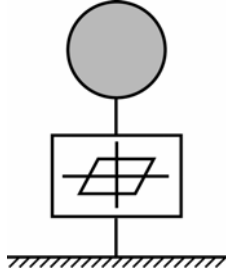


Fig. 1: SDOF system with nonlinear hysteresis.

It is well known that the fundamental seismic response behavior of a base-isolated building can be practically estimated by using a nonlinear single-degree-of-freedom (SDOF) dynamical system model like Fig. 1. For such a SDOF seismic-isolated structural system, the mass represents a mass of the upper structure above an isolator system exhibiting nonlinear hysteresis. The behavior of the structural system having a mass of m is represented by

$$m\ddot{y} + Q(y, \dot{y}) = -m\ddot{x}_g \quad (1)$$

in which y = displacement relative to the base; $Q(y, \dot{y})$ = nonlinear restoring force resulting from the isolation system; and \ddot{x}_g = ground surface acceleration due to an earthquake.

This nonlinear equation is equivalently linearized with two parameters of linearized stiffness and damping, k_{EQ} and c_{EQ} , in the following fashion:

$$m\ddot{y} + c_{EQ}\dot{y} + k_{EQ}y + \varepsilon = -m\ddot{x}_g \quad (2)$$

in which ε represents the resulting residue from the linearization. One of the techniques properly deter-

mining the two key parameters in the above linearized equation is that they are determined in such a way as to minimize the mean square of the resulting residue. With the mean square of the residue denoted as $E[\varepsilon^2]$, the conditions for determining the linearized parameters are:

$$\partial E[\varepsilon^2] / \partial k_{EQ} = 0, \quad \partial E[\varepsilon^2] / \partial c_{EQ} = 0 \quad (3)$$

From the above conditions,

$$c_{EQ}E[y\dot{y}] + k_{EQ}E[y^2] - E[yQ(y, \dot{y})] = 0 \quad (4)$$

$$c_{EQ}E[\dot{y}^2] + k_{EQ}E[y\dot{y}] - E[\dot{y}Q(y, \dot{y})] = 0 \quad (5)$$

For the case the system is excited by a stationary process, the linear response also becomes stationary, i.e. $E[y\dot{y}] = 0$. In this case, the linearized stiffness and damping is given by:

$$k_{EQ} = E[yQ(y, \dot{y})] / E[y^2] \quad (6)$$

$$c_{EQ} = E[\dot{y}Q(y, \dot{y})] / E[\dot{y}^2] \quad (7)$$

The procedure based on the least square method, which is briefly presented in the above, is called "Dynamic Stiffness Method" (DSM) (Jennings 1968).

2.2 Steady-state response of bilinear systems

Consider a SDOF system exhibiting a type of bilinear hysteresis depicted in Fig. 2. This kind of hysteresis is one of the most typical ones for base isolation systems consisting of rubber bearings and dampers. Consider the case in which this nonlinear system is excited by steady-state sinusoidal acceleration, i.e. $\ddot{x}_g = a \cos pt$. By assuming that such an excitation results in steady-state response, the equation becomes:

$$m\ddot{y} + Q(y, \dot{y}) = -ma \cos pt \quad (8)$$

with

$$y = R \cos(pt - \varphi) \quad (9)$$

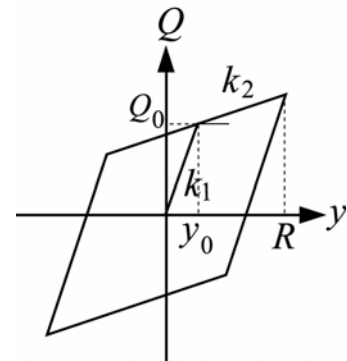


Fig. 2: Bilinear hysteresis.

The use of this type of solution has been employed in Caughey (1960).

The required parameters for constructing the linearized model are obtained based on the basically same procedure presented in the previous subsection, utilizing the temporal averages in estimating $E[y]$ and $E[\dot{y}]$. As a solution of the type like Eq.(9) has been assumed, $E[y \dot{y}]$ becomes zero. Then,

$$k_{EQ} = \int_{-T/2}^{T/2} y Q(y, \dot{y}) dt / \int_{-T/2}^{T/2} y^2 dt \quad (10)$$

$$c_{EQ} = \int_{-T/2}^{T/2} \dot{y} Q(y, \dot{y}) dt / \int_{-T/2}^{T/2} \dot{y}^2 dt \quad (11)$$

For the dynamical structural system with the bilinear hysteresis shown in Fig. 2, the above formulations give the linearized parameters in the following fashion:

$$k_{EQ} = \frac{1}{\pi} [(k_1 - k_2)\theta^* + k_2\pi - \frac{k_1 - k_2}{2} \sin 2\theta^*] \quad (12)$$

$$c_{EQ} = \frac{4(k_1 - k_2)}{\pi p} \cdot \frac{y_0}{R} \left(1 - \frac{y_0}{R}\right) \quad (13)$$

with

$$\theta^* = \cos^{-1} \left(1 - 2 \frac{y_0}{R}\right) \quad (14)$$

The above results have been presented by Caughey (1960). These linearized parameters given in the above clearly depend upon the maximum response value, R . Thus, the resulting linearized solution based on the employment of such parameters should satisfy:

$$R \approx |y(t)|_{\max} \quad (15)$$

The parameters for linearization are repeatedly tested until the resulting response agrees with the assumed maximum response.

2.3 Seismic responses of base-isolated buildings with bilinear systems

Even in case a stochastic process excites the structure discussed in the preceding subsection, it is assumed that a similar solution to Eq.(9) for sinusoidal excitation represent the dynamical behavior, which is:

$$y = R \cos(\omega_{EQ} t - \varphi) \quad (16)$$

with

$$\omega_{EQ} = \sqrt{k_{EQ}/m} \quad (17)$$

In this case, the amplitude, R , and phase angle, φ , change with time. However, it is assumed that, com-

pared to the resulting natural period, they change their values relatively slowly.

Following the procedure employed for the case of sinusoidal excitation, the two parameters are provided by:

$$k_{EQ} = \frac{1}{\pi} [(k_1 - k_2)\theta^* + k_2\pi - \frac{k_1 - k_2}{2} \sin 2\theta^*] \quad (18)$$

$$c_{EQ} = \frac{4(k_1 - k_2)}{\pi} \cdot \sqrt{\frac{m}{k_{EQ}}} \cdot \frac{y_0}{R} \left(1 - \frac{y_0}{R}\right) \quad (19)$$

in which θ^* is the same as Eq.(14).

Fig. 3 demonstrates how the resulting equivalent period and damping ratio from Eqs.(18) and (19) change with the increase of ductility factors for a model with a initial natural period of 1(s) and no original viscous damping. It is recognized that the equivalent period becomes a constant value as the ductility factor increases, while the equivalent damping ratio tends to decrease with the increase of deformation.

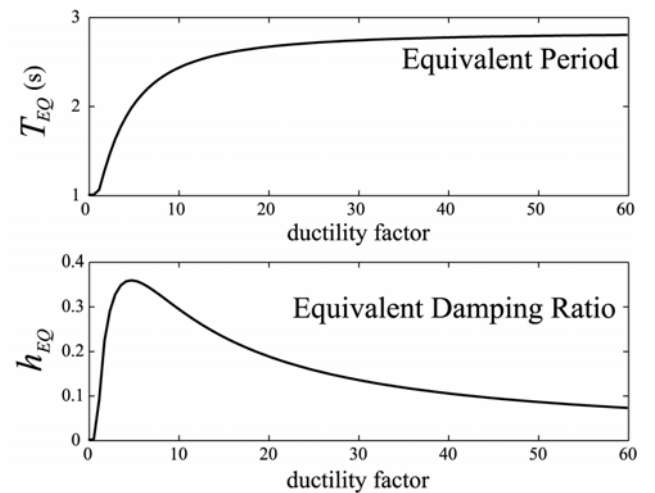


Fig. 3: Resulting natural periods and damping ratios.

For the purpose of demonstrating the validity of the above equivalent linearization technique based on the least square method, the seismic responses of a base-isolated building are estimated by means of the DSM technique. Assume that a SDOF structure modeling a base-isolated building has the following parameters related to the hysteresis loop like Fig. 2: $m = 10,000$ (ton); $k_1 = 394,780$ (kN/m); $k_2 = 43,865$ (kN/m); $Q_0 = 2,940$ (kN). These parameters are set in such a way that the initial natural period be 1.0 (s) and the natural period after the yielding be 3.0 (s). This example model is subjected to seismic excitations of the EW component of the 1968 Hachinohe earthquake with two peak ground velocities of 0.1 (m/s) and 0.5 (m/s). Fig.4 presents the acceleration responses. Both of the two responses calculated by means of the DSM technique are compared with the results based on nonlinear analyses. It is found that

the peak response value to the severe seismic excitation agrees quite well with the result from the nonlinear analysis, while that to the small excitation does not do very well. In regard to the general form of time histories, the linearized response satisfactorily agrees with the nonlinearly analyzed history in the case of severe seismic excitation, in particular when exhibiting the large amplitudes, while the case of small excitation does not give such an agreement. The above tendency indicates that the formulation based on the steady state sinusoidal response provides us with the idea how an assumed or designed base-isolated building behaves in case of severe earthquake.

Unlike most of the general non-isolated buildings, base-isolated buildings are likely to have large values of “ductility factors.” In a base-isolated building exhibiting a hysteresis like Fig. 2, the displacement and force corresponding to the yield points of normal buildings are relatively small, and hence the resulting ductility factor, estimated as the maximum displacement divided by the yielding displacement, tend to represent large values. In this regard, the response analysis of base-isolated buildings may be well fitted to the use of the equivalent linearization technique.

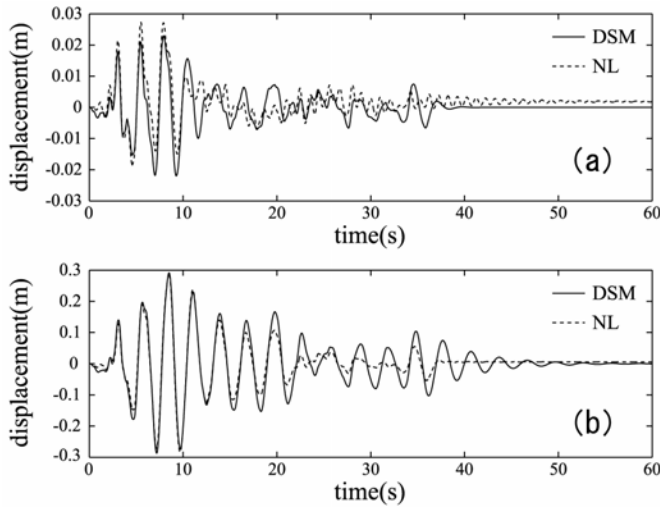


Fig.4: Comparison of linearized and nonlinear-analyzed results for the 1968 Hachinohe (EW) earthquake with a peak velocity of (a) 0.1 m/s and (b) 0.5 m/s.

3. PERTURBATION APPROACH

3.1 Treatment of nonlinear systems

It is well known that the mechanical properties of rubber bearings and dampers integrated into base-isolated buildings have some fluctuations. At the stage of practical design of base-isolated buildings, these fluctuations are accounted for. In most cases, however, the treatment of those mechanical property

fluctuations is rather on the deterministic basis than on the probabilistic basis.

There are several probabilistic-based approaches handling the fluctuations involved in nonlinear dynamical structural systems. Monte Carlo simulation approach is one of the most widely utilized schemes dealing with such fluctuations. This approach can be employed for either linear systems or nonlinear systems. From the practical design point of view, however, Monte Carlo simulations may need too much calculation if the results thus simulated are somehow reflected in the actual structural design. The approach based on the Fokker-Planck equation has severe restrictions if aiming at the use of it at the practical design stage, although it provides an exact solution.

The perturbation method, on the other hand, can be effectively utilized in combining with the equivalent linearization method. The perturbation approach could be applied to either nonlinear or linear problem. In applying this approach to nonlinear system problems, the response resulting from nonlinearity is treated as the fluctuation from the linear system response. For this reason, a weakly nonlinear problem is appropriate for this approach. In this paper, the perturbation method is applied to the equivalently linearized system in combination with the equivalent linearization method (DSM). The basic scheme is presented in the following subsection.

3.2 Formulation in combination with linearization technique

Consider an equivalent-linearized SDOF system, which represents the behavior of a base-isolated building exhibiting nonlinear hysteresis in response to seismic excitation. In a base isolation system there are several fluctuations involved in such parameters as the initial stiffness of damper, the yielding force of damper, the stiffness of rubber bearing, etc. The resulting linearized stiffness and damping values are dependent of the response and the response are dependent of those fluctuating parameters, denoted as X_1, X_2, \dots . The linearized equation of motion is given by:

$$m\ddot{y} + c_{EQ}\dot{y} + k_{EQ}y = -m\ddot{x}_g \quad (20)$$

In the above equation, the resulting linearized coefficients are expanded in powers of the fluctuating parameters in the perturbation approach (Iwan & Yang 1972). Following the perturbation approach based on the first-order approximation, the linearized stiffness and damping are expanded up to the first power of these parameters in the following fashion:

$$k_{EQ} = k_{EQ}^0 + k_{EQ,i}^1 \varepsilon_i \quad (21)$$

$$c_{EQ} = c_{EQ}^0 + c_{EQ,i}^1 \varepsilon_i \quad (22)$$

in which

$$\varepsilon_n = X_n - E[X_n] \quad (23)$$

$$E[k_{EQ}] = k_{EQ}(E[X_n]) = k_{EQ}^0 \quad (24)$$

$$E[c_{EQ}] = c_{EQ}(E[X_n]) = c_{EQ}^0 \quad (25)$$

$$k_{EQ,n}^1 = \partial k_{EQ} / \partial X_n \quad (26)$$

$$c_{EQ,n}^1 = \partial c_{EQ} / \partial X_n \quad (27)$$

In the above formulation, the summation convention has been employed for simple expressions, thus with the repeated subscripts indicating summation. The summation convention will be employed in the succeeding sections as well.

In this first-order approximation approach of the perturbation method, the solution of the linearized equation of motion, Eq.(20), is assumed to be given by:

$$y = y^0 + y_i^1 \varepsilon_i \quad (28)$$

Substituting the assumed solution into Eq.(20) with the relations of Eqs.(21) and (22) accounted for and equating coefficients of the same order of powers of the fluctuating parameters then yields the following formulation:

$$m\ddot{y}^0 + c_{EQ}^0 \dot{y}^0 + k_{EQ}^0 y^0 = -m\ddot{x}_g \quad (29)$$

$$m\ddot{y}_i^1 \varepsilon_i + c_{EQ}^0 \dot{y}_i^1 \varepsilon_i + k_{EQ}^0 y_i^1 \varepsilon_i = -(c_{EQ,i}^1 \varepsilon_i \dot{y}^0 + k_{EQ,i}^1 \varepsilon_i y^0) \quad (30)$$

By equating the both sides' coefficients for each of ε_n in Eq.(30),

$$m\ddot{y}_n^1 + c_{EQ}^0 \dot{y}_n^1 + k_{EQ}^0 y_n^1 = -(c_{EQ,n}^1 \dot{y}^0 + k_{EQ,n}^1 y^0) \quad (31)$$

Then, y_n^1 in Eq.(28) is obtained for each of ε_n and the expected response is:

$$E[y] = y^0 + y_i^1 E[\varepsilon_i] = y^0 \quad (32)$$

The variance is given by:

$$Var(y) = E[(y - y^0)^2] = y_i^1 y_j^1 E[\varepsilon_i \varepsilon_j] \quad (33)$$

In the case in which each of ε_n is uncorrelated each other, the variance leads to:

$$Var(y) = E[(y - y^0)^2] = (y_i^1)^2 E[\varepsilon_i^2] \quad (34)$$

4. NUMERICAL EXAMPLES

For the purpose of demonstrating the validity of the methodology combining the equivalent linearization and perturbation techniques, the results of some numerical examples are presented.

For the same model structure as employed in the example of Fig. 4, Fig. 5 gives the mean value and standard deviation of the response estimated on the basis of the combined method of DSM and perturbation approach, assuming that the structure involving the fluctuation of the second stiffness k_2 with a coefficient of variation of 10% is excited by the 1968 Hachinohe (EW) earthquake with a peak velocity of 1.0 (m/s). It can be realized that the proposed method estimates larger standard deviations for larger mean responses.

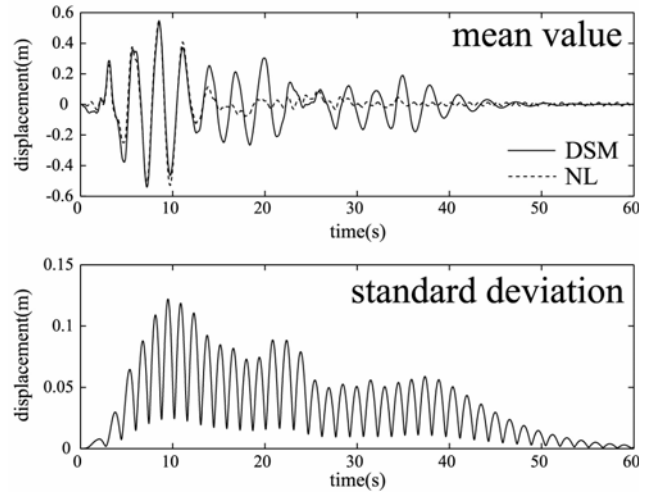


Fig. 5: Mean and standard deviation of response estimated by the combined method of DSM and perturbation technique.

To realize how accurate an estimation the proposed method gives for different magnitudes of seismic excitation, the coefficient of variation of the maximum displacement are estimated by the proposed method considering a coefficient of variation of 10% for the second stiffness and no fluctuation for other parameters. Based on the formulation given by Eqs.(32) and (33), the coefficients of variation of the response are evaluated for the 1968 Hachinohe (EW) and 1995 JMA Kobe (NS) earthquakes with the peak velocities changing from 0.1 (m/s) to 1.2 (m/s).

Table 1 compares the estimated results of the coefficient of variation with those from the Monte Carlo simulation (MCS). In conducting the Monte Carlo simulation, 5000 sample structures with different values of log-normal-distributed second stiffness have been generated for each of the magnitude of the earthquake peak velocity. For different seis-

mic excitations, the totally different results in terms of the magnitude of coefficient of variation are obtained by the proposed method, although the results are in relatively good agreement with the MCS results.

Table 1: Estimated coefficients of variation of the maximum displacement responses.

Peak velocity (m/s)	Hachinohe (EW)		JMA Kobe (NS)	
	MCS	Perturbation	MCS	Perturbation
0.1	0.26	0.24	0.65	0.07
0.2	2.19	0.44	1.38	0.12
0.3	2.83	1.19	2.43	0.20
0.4	1.17	2.36	3.28	0.31
0.5	4.08	3.99	0.30	0.42
0.6	8.68	5.73	0.41	0.54
0.7	13.47	7.53	0.72	0.67
0.8	14.20	9.25	0.85	0.78
0.9	14.11	11.05	0.77	0.89
1.0	13.55	12.95	0.86	0.99
1.1	13.17	14.74	0.94	1.08
1.2	12.99	16.41	1.05	1.16

5. CONCLUSIONS

Base isolation systems consisting of rubber bearings and dampers exhibit nonlinear hystereses during a seismic event. In consideration of the fact that the parameters involved in such hystereses are likely to fluctuate, the nonlinear vibration of base-isolated buildings with such fluctuations has been estimated by utilizing the methodology combining the equivalent linearization technique and perturbation method. The resulting responses satisfactorily give the information about the peak responses as the first step entering into the detailed design. The presented methodology can be employed in the stage of practical design of base-isolated buildings.

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