1. Introduction

1.1 Definition

- A process to develop or improve mathematical representation of a structural system using experimentally obtained structural responses.

- Mathematical representation of a structural system: Mass, Stiffness, Damping, Flexibility, Connectivity

- Experimentally obtained structural response: vibrations, static response/deflection, strain response etc.

- System identification is a broad term, when ‘system’ refers to structural system, the term structural identification is commonly used.

1.2 Objectives

Why System Identification for constructed structures?

1. Model Validation of newly constructed structures
   - verify assumptions in design model (e.g., boundary condition, nonlinear behavior, energy dissipation mechanism/damping)
   - verify performance of control system (e.g., base-isolation, Tuned Mass Damper, etc.)

2. Model Updating
   - obtain FEM calibrated structural model
   - adjust structural parameters after retrofit or modification

3. Structural/Condition Assessment and Health Monitoring
   - detect structural changes possibly due to defect or damage
   - recognize environment/loading influence or pattern on the structure
1. Introduction

Example of Local Structural Identification: Evaluation of damping on stay cable of cable-stayed bridge to assess the effectiveness of cable damper system.

2. Exp Methods | Classification of Required Data

The required data is data collected during experiment and can be classified into:

- **Excitation**: measurements made of disturbance forces, pressure, impact, stress applied to the structure.
- **Response**: measurements made of the reactions of the structures to the applied disturbance, such as, deflection, displacement, velocity, acceleration, strain etc.

3. Exp Methods | Type of Excitation

The excitation can be classified as:

1. Dynamic or static (i.e. according to whether or not they engage inertial effects)
2. According to controllability, and
3. According to measurability

- Controllable (measurable and un-measurable) static loads
- Uncontrollable (measurable and un-measurable) static loads
- Controllable (measurable and un-measurable) dynamic loads
- Uncontrollable measurable dynamic loads
- Uncontrollable un-measurable dynamic input (ambient dynamic excitation)

4. Exp Methods | Static Loads

Controllable (measurable and un-measurable) static loads

Relatively rare for full-scale experiments on real structures because of the scale of the load required to generate a measurable effect.

Common example is proof testing of bridges often involving use of heavy vehicles, either stationary or moving

Uncontrollable (measurable and un-measurable) static loads

Generally include elements of dynamic load and response monitoring, particularly in the case of traffic and wind which generate quasi-static and dynamic response.

5. Exp Methods | Dynamic Loads – Controllable Measurable

- Forced vibration test (FVT)
  Transfer functions or frequency response functions (FRFs) scale input (forcing) to output (response) via either mass or stiffness so can both be identified, along with high quality information about dissipative effects (mathematically realised as viscous damping)

Examples: mass exciters, Electro-dynamic shakers, instrumented hammer


- Manual excitation
  Impulse response functions (IRF) or free-vibration response. Neither mass nor stiffness can be identified. Modal frequency and damping can be estimated quite accurately.

Examples: Impact hammer, people jump, drop weight test, Snap-back or or step relaxation test
### 2. Exp Methods: Dynamic Loads – Controllable Un-Measurable Excitation

**Controllable but unmeasurable dynamic loads**

- Manual excitation: Impact Hammer Test

![Image: Free vibration response of the bridge subjected to impact hammer]

Giving excitation to a short span bridge by impact hammer

- Manual excitation: Drop Weight Test

![Image: Example of Free vibration response of the bridge excited by dropped weight]

Giving excitation to a short span bridge by dropping sand bag weight

**Note:** while drop weight test is effective in exciting the free-vibration response of the structure, additional damping is expected as the dropped weight tends to increase the damping.

### 2. Exp Methods: Dynamic Loads – Controllable Un-Measurable Excitation

**Manual excitation: Pull-and-released test of stay cable**

![Image: Flowchart to obtain damping value of a stay cable]

Giving excitation to a stay cable by pull-and released test

### 2. Exp Methods: Dynamic Loads – Uncontrollable Measurable Excitation

**Seismic excitation**

Transfer functions or frequency response functions (FRFs) between seismic input (base excitation) to output (structure response). Structural properties, modal properties and modal participation factor can be estimated.

**Example:** instrumented bridges and buildings in Japan and California US.

![Image: Example: Yokohama Bay Bridge, instrumented cable-stayed bridge near Tokyo]

### 2. Exp Methods: Dynamic Loads – Uncontrollable Un-measurable Excitation

**Ambient excitation: wind, traffic, and unmeasured micro-tremor**

Correlations between response are used to estimate modal properties. Mode-shapes unscaled. Treated as stochastic system identification.

**Example:** periodic ambient vibration measurement and instrumented bridges and buildings.
Example of ambient excitation: wind-induced vibration of suspension bridge.

Tower acceleration response

Treated as stationary random process.

Example of ambient excitation: traffic-induced vibration of bridge.

Bridge response subjected to open traffic usually treated as stationary random process, since the input is unknown. Effect of vehicle mass is usually neglected. However, in case of short span bridge, the effect of vehicle mass may not be negligible and influence the identified bridge frequency.

Example of vertical acc and the spectrum of a medium span highway bridge to traffic.

Static: Strain, Deflection

Dynamic: Acceleration, Relative of absolute displacement, Velocity, Inclination, Strain, Stress, Water Pressure, Structural and environmental temperature, Wind Velocity, Wind Direction

Examples of Parametric Model

Output-Error Minimization for system identification using seismic response.

By modeling the input-output relationship of seismic-induced vibration using state-space model and realization of observability matrix, system matrices A, B, R and D can be obtained and modal parameters are realized.
3. Analysis Methods

### Types of Model

#### 1. Structural Model
- System is modeled in terms of mass, stiffness, or flexibility, and damping matrices.
- Geometric distribution of mass, stiffness and damping are known.
- Structural connectivity between degree of freedom is preserved.

**Equation of motion:**

\[ M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = -Rz(t) \]

System matrix A in state-space form for discrete data:

\[ A = \begin{bmatrix} 0 & 1 \\ -M'' & -J'' \end{bmatrix} \]

**Equation of motion in discrete dynamic system, where system matrix A is to be identified**

\[ x(k+1) = \begin{bmatrix} A \\ \end{bmatrix} x(k) + \begin{bmatrix} \end{bmatrix} z(k) \]

\[ y(k) = \begin{bmatrix} R \end{bmatrix} x(k) + \begin{bmatrix} D \end{bmatrix} z(k) \]

**Goal:** To indentify system matrix A in its original form, from which the mass, stiffness and damping matrices can be retrieved.

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#### 3. Analysis Methods

### Non-Physical Numerical Model

- Does not have physical relationship with the structure (i.e., no spatial information, no geometry distribution of mass, stiffness and damping).
- Simply a parameter curve-fit of the given mathematical model to the measured data.
- Examples: Auto Regressive Moving Average (ARMA) and its variants, Rational Polynomial Model etc.
- Some can be converted to modal model form.

**Example:** Auto Regressive Moving Average (ARMA) Model where the auto regressive coefficients can be related to modal parameters

\[
\frac{d^n y(t)}{dt^n} + a_1 \frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots + a_n y(t) = b_0 \frac{d^n u(t)}{dt^n} + b_{n-1} \frac{d^{n-1} u(t)}{dt^{n-1}} + \ldots + b_1 u(t)
\]

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### Domain

#### 1. Frequency Domain
- Peak Picking Method
- Transfer Function / Frequency Response Function / Impulse Response Function.
- Average Normalized Power Spectrum Density (ANPSD)
- Complex Exponential Frequency Domain Method (Schmerr 1982)
- Eigensystem Realization Algorithm in Frequency Domain (ERA-FD) (Juang & Pappa 1985)
- Frequency Domain Decomposition (Brincker et al. 2001)

#### 2. Time Domain
- Ibrahim Time Domain (ITD) (Ibrahim & Mikulcik 1973)
- Least Squared Complex Exponential Method (LSCE) (Brown 1979)
- Polynomial Complex Exponential Method (PCF) (Vold et al 1982)
- Eigensystem Realization Algorithm (ERA) (Juang & Pappa 1985)
- Stochastic Subspace Identification (Overschee & De Moor 1991)

#### 3. Cross Time-Frequency Domain
- Represents frequency evolution as time progresses.
- Can detect non-linearity and non-stationary signals
  - Short Time Fourier Transform (STFT)
  - Wavelet-based system identification
  - Empirical Mode Decomposition – Hilbert Huang Transform (should be carefully applied since EMD lacks physical meaning of signals)
3. Analysis Methods | Direct and Indirect Method Time Domain

**Direct Method**
When the IRF/FRF is available, they can be used as input directly to system identification method.

**Indirect Method**
When the IRF/FRF is unavailable such as in case of ambient vibration measurement, an additional method is needed to construct synthetic IRF, ex. through cross-correlation (Natural Excitation Technique (NEXT)) or through Random Decrement.

Input:
- Raw Data
- NEXT
- ERA
- Randoc
- ITD

4. Uncertainties

Uncertainty is unavoidable in understanding the results of system identification. Modal properties are susceptible to variation even when structural condition remains the same.

How to quantify the confidence of the identified modal properties?
1. Error propagation analysis using perturbation method
2. Monte Carlo Simulation
3. Bootstrap Method

Bootstrap Analysis

Example of investigation of the effect of variability and to estimate the confidence bounds of identified modal parameters by NEXT-ERA

CCF : Cross-correlation function

4. Uncertainties | Error propagation analysis using perturbation

Example of error propagation in System Realization using Information Matrix

Information Matrix

Realization of System Matrix:

Realization of Modal Parameters:
- \( \delta(0) = \delta(0) + \delta(0) \)
- \( \xi(0) = \xi(0) + \xi(0) \)
- \( \zeta(0) = \zeta(0) + \zeta(0) \)

Objectives:
To define and quantify the error on the modal parameters as the effect of input and output noise

4. Uncertainties | Bootstrap Analysis

Distribution of modal parameters and their mean values and confidence level can be obtained

Modal parameters are considered as stochastic variables that have distribution with certain statistical characteristics. Therefore, decisions made on structural condition involve statistical confidence.
5. Examples: Ambient Vibration Measurement of Suspension Bridge

Structural identification: effect of friction force and aerodynamic forces on identified frequency and damping (Nagaya et al. 2005)

5. Examples: Seismic-Induced System Identification of Cable-Stayed Bridge

A data driven identification method was applied considering multiple input excitation and multiple responses (MIMO System)

5. Examples: Seismic-Induced System Identification of Cable-Stayed Bridge

With dense instrumentation and good quality of seismic records we identify bridge modal parameters until high order
5. Examples: Seismic-Induced System Identification of Cable-Stayed Bridge

Observation of the performance of seismic isolation devices using 1st longitudinal mode (Siringoringo & Fujino 2008)

From the first longitudinal mode we can observe behavior of Link-Bearing Connection during earthquake.

Different behavior of Link-Bearing Connection at the end-piers was observed during different level of earthquake excitation.

It was found that the expected slip-slip mode only occurred during large earthquake.

Suggested Readings Materials:

- Theoretical and Experimental Modal Analysis by Maia, Silva, He, Lieven et al.
- Applied System Identification by Jer Nan Juang
- Monitoring and Assessment of Structures by GST Armer

Q & S

Questions and Sharing?

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